THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXVII OCTOBER 1908 NUMBER 3

WAVE-LENGTH MEASUREMENTS FOR THE ESTABLISHMENT OF A SYSTEM OF SPECTROSCOPIC STANDARDS[1]

By C. FABRY and H. BUISSON

I. INTRODUCTION

The researches about to be described were undertaken as a result of a decision of the Congress of the International Union for Solar Research held at Oxford in 1905. In accordance with this decision, the wave-lengths of a certain number of arc lines were to be taken as standards for spectroscopic measurements. Briefly let us recall the considerations which led to this.

All spectroscopic measurements (determinations of the wave-lengths of the light corresponding to various lines of the spectrum) are carried out by interpolation, the wave-length of any line being expressed in terms of the known wave-lengths of neighboring lines. In order to make these measurements it is thus necessary to know the wave-lengths of a certain number of lines (standards, or normal lines). It is desirable that these be sufficiently numerous to render interpolation always easy.

A group of standards constitutes what may be called a system of wave-lengths. Its precision limits that of all other spectroscopic measurements. But the powerful dispersive apparatus at present

[1] Translated from the authors' manuscript by Professor R. R. Tatnall of Northwestern University.

available (Rowland gratings or prism-trains) admits of making, by interpolation, measurements of relative wave-lengths accurate to one part in a million. Hence it is necessary that the wave-lengths of the standards shall be known with an accuracy of at least one part in a million.

From a purely spectroscopic point of view, wave-lengths appear only as ratios; we might express them in terms of an arbitrary unit. But since wave-lengths are actual lengths, it is evidently reasonable to refer them to the ordinary unit of length, the meter. The comparison of a wave-length with the meter is a metrological process, requiring special methods, and demanding the use of the material standard which represents the meter, while the comparison of two wave-lengths is a purely optical process. The establishment of a system of wave-lengths thus comprises two distinct parts: (1) comparison of a wave-length of a certain radiation with the meter; (2) measurement of the ratios of the wave-length of this radiation to those of a certain number of others which are to serve as standards in the spectrum.

The older systems of wave-lengths, such as that of Ångström, did not aim at a high degree of precision; the values were given to five figures only, that is, with an accuracy of less than one in a hundred thousand, an accuracy no doubt sufficient in view of the small dispersion of the apparatus then employed.

During the last twenty years, all spectroscopic measurements have taken Rowland's numbers as a starting-point, although other observers have, during the same period, given results of at least equal precision. The numbers given by Rowland comprise all the lines which he was able to observe in the solar spectrum (about 20,000) and a small number of metallic lines derived from arc spectra. The absolute value constituting the basis of the system was the wave-length of the D_1 line obtained by Bell in Rowland's laboratory. A certain number of lines were compared with this by the method of coincidences, and the others were determined by interpolation.

Rowland believed that the absolute values were correct to one part in one hundred thousand, and that the ratios of the wave-lengths should not contain errors exceeding one part in a million.

The absolute measurements of Michelson and Benoit, in 1893, upon cadmium lines, showed that Rowland's values were in error by

about 1 in 30,000. Spectroscopists paid little attention to this result. They were interested only in the ratios of wave-lengths, and there was no reason to doubt the relative accuracy of Rowland's values. In 1901, Fabry and Perot[1] made a series of interferential measurements upon solar lines. Some thirty lines between λ 4643 and λ 6471 were compared separately with one of the lines of cadmium. Let λ be the exact wave-length thus determined, and λ_1 the value given by Rowland for the same line. If Rowland's numbers are correct in relative value, the ratio $\frac{\lambda_1}{\lambda}$ should be the same for all lines. This is not the case. The ratio varies from one line to another, and it varies in a regular manner in that part of the spectrum which was studied. For two lines which are near each other this ratio has sensibly the same value; for two distant lines the discrepancy may reach eight parts in a million. Thus, although the accidental errors in Rowland's tables are very slight, there are systematic errors which considerably affect the ratios of the wave-lengths.

These errors may be attributed to the use of gratings, which inevitably show systematic errors of ruling. Moreover, Kayser[2] has undertaken to repeat the wave-length comparisons by the coincidence method, using Rowland gratings. He has found that the results depend upon the gratings employed.

We may conclude that the grating, which is an excellent dispersive piece, is well adapted for measurements made by interpolation within a narrow interval, but is unsuitable either for absolute measurements or for the comparison of widely separated lines.

It should further be stated, in regard to the use of Rowland's standards, that they are based upon solar lines. But one nearly always employs in standard measurements, not solar light, but that of an artificial source, such as the arc or spark. Many lines of these sources correspond to dark lines of the solar spectrum, but there is no absolute identity of wave-length. It is true that Rowland has given the wave-lengths of a certain number of arc lines, yet these lines are comparatively few, and their wave-lengths are subject to uncertainty,

[1] *Annales de Chimie et de Physique* (7), **25**, 98, 1902; *Astrophysical Journal*, **15**, 73 and 261, 1902.

[2] *Astrophysical Journal*, **19**, 157, 1904.

because they were measured with less care than the solar lines, and with a preconceived idea that the differences between the solar lines and the arc lines ought to be zero or exceedingly small.

In conclusion, the following reasons are sufficient for the rejection of Rowland's system: the absolute values are wrong by one part in 30,000; the relative values show systematic errors amounting to nearly 1 in 100,000; and finally, the measurements are based upon the solar spectrum, while metallic spectra are usually employed for purposes of comparison.

This question was the subject of several reports presented at the first meeting of the International Union for Solar Research.[1] Most authors (Crew, Perot and Fabry, Kayser) agreed as to the necessity of establishing a new system of wave-lengths, derived from artificial sources. No decision was reached at this meeting, which was, in a sense, preliminary.

The second meeting (Oxford, 1905) took up the question anew, and arrived at the following decisions:[2]

1. The wave-length of a suitable spectroscopic line shall be taken as the primary standard of wave-length. The number which defines the wave-length of this line shall be fixed permanently and thereby define the unit in which all wave-lengths are to be measured. This unit shall differ as little as possible from 10^{-10} meter, and shall be called the Ångström.

2. Secondary standards are required at distances not greater than 50 Ångström units. These secondary standards should be referred to the primary standard by means of an interferometer method. The source of light should be obtained by means of an electric arc of from 6 to 10 amperes.

The adoption of arc lines, in preference to solar lines, to serve as standards, is justified by the following considerations: an artificial source is more convenient to use; one has it always at hand; in most cases bright lines are more serviceable than dark lines; and finally, the wave-lengths of solar lines are affected by the motions of the sun and of the earth, and perhaps also by phenomena

[1] *Astrophysical Journal*, 20, 301, 1904.

[2] *Transactions of the International Union for Co-operation in Solar Research*, 1, 230, 1906.

peculiar to the sun, which are not constant, and which cannot be controlled.

The maximum interval of 50 Ångströms between two consecutive lines requires the determination of about one hundred standards throughout the whole of the visible and ultra-violet spectra. This large number of standards is necessary; with spectroscopes of large dispersion only a small portion of the spectrum can be observed at one time, and this should contain several standards. The necessity is still more marked in the case of prism spectroscopes, in which the formulas of interpolation are not simple.

In 1907, a third meeting took place at Meudon. It was then possible to reach a definite decision in regard to the choice of a primary standard. The determinations of Benoit, and Fabry and Perot, in almost rigorous accordance with those of Michelson and Benoit, give for the wave-length of the red cadmium line the value $6438.4696 \times 10^{-10}$ meter in dry air at 15° C. and under normal pressure. It was decided to adopt the red cadmium line as the primary standard, and to designate its wave-length by the number 6438.4696 Ångströms. This is the definition of the Ångström; it may be considered as certain that it is equal to 10^{-10} meter with an accuracy of about one part in ten million.

Our work has for its object the determination of the secondary standards. Before deciding upon these, it was desirable that independent measurements should be made by several persons. At the time of the Meudon meeting, our investigation was the only one completed, so that no decision could then be reached.

II. SOURCES OF LIGHT

The cadmium light, which is to serve as primary standard, is produced by a Michelson tube, fed by continuous current from a battery of small accumulators having a voltage of about 1200; the current, which is of a few milliamperes, is regulated by a liquid resistance.

Instead of employing cadmium light for the direct comparisons with other radiations, we have preferred to adopt an intermediary standard, which is more convenient to use, and better situated in the spectrum, from a photographic standpoint. For this purpose

we have used the Cooper-Hewitt mercury vapor lamp, which gives a very intense light. We have chosen the green line at λ 5460 for the intermediary standard. A cell containing normal potassium chromate, and one of didymium chloride absorb all other radiations; the didymium cell being unnecessary if the photographic plates used are but slightly sensitive to the yellow.

The lines which we have measured are produced by means of the electric arc, and chiefly by the iron arc. This arc was formed between two iron rods 7 mm in diameter, placed vertically, and in a holder for hand-regulation. The use of an automatic feed would have resulted in nothing but inconvenience, since the rate of consumption of the iron rods is very small. The current, which should be continuous, was usually of from 3 to 5 amperes. On increasing the current strength, the lines widen out, and the measurements become less accurate, and with some lines even impossible. The arc is extremely stable, provided the current is not too strong, nor the voltage of the source too low. This source was most commonly a distributing system at 110 volts, in which was inserted an adjustable resistance of some twenty ohms. Under these conditions, the arc was so stable that it was possible to make photographic exposures of half an hour without touching it. In exceptional cases we have made use of the same current strength under 220 volts pressure. One may thus obtain an arc, which, while very stable, is longer, so that the light emitted by different parts of the arc may be more readily separated.

In the choice of lines to be measured, one is compelled to keep within the interval of 50 Ångströms between consecutive lines. Most of the very strong lines were not measured because they were not sharp enough. All those were included which had been measured by Fabry and Perot. In certain parts of the spectrum, lines available as standards are lacking. We have measured two lines of manganese which are always present with sufficient intensity in the spectrum of the arc between ordinary iron rods. In the neighborhood of λ 5800 we have measured four nickel lines given by the arc playing between two rods of that metal; this arc being as stable as that of iron. Finally, in the extreme ultra-violet, the iron lines become, for the most part, very faint; and we have measured three

silicon lines formed by the arc between carbons, such as are used for illumination. In this region, the continuous spectrum due to the carbons no longer exists; there is nothing but the light emitted by the gaseous part, and the lines of silicon are the brightest.

III. METHOD

The method employed admits of comparing each radiation with the same fundamental radiation, so that the relative measurements upon various lines are completely independent of one another. This method is the interferential one, depending on the use of silvered surfaces. Although it has already been described in detail, a brief description will be repeated here.

The interference apparatus consists of two plates of glass or of quartz, each having one silvered surface. These surfaces are put face to face, and brought into exact parallelism. They inclose between them an air-film of uniform thickness, e.

Let us suppose this apparatus illuminated by a source of monochromatic light of wave-length λ, and let us observe with a telescope focused upon infinity. To each point of the field corresponds a definite direction of the rays, and consequently a determinate angle of incidence at the air-film. For a given point, M, of the field, let the angle of incidence be i. This point M receives one ray which has traversed the apparatus, one which has suffered two reflections at the faces of the air-film, one which has suffered four reflections, and so on. The differences of path of these several rays with respect to the first ray will form an arithmetical progression whose difference is $2e \cos i$. The order of interference at the point M will be $\frac{2e \cos i}{\lambda}$.

We see from this that the interference phenomenon takes the form of rings with their centers at that point of the field which corresponds to normal incidence upon the silvered surfaces, i. e., the point for which $i=0$. The bright rings are fine lines, separated by wide dark intervals, and this characteristic becomes more marked as the reflecting power of the surfaces increases. Each ring is characterized by an integral order of interference, which diminishes by unity from one ring to the next, from the center outward. The

angle of incidence has the same value for all the points of any one ring; we shall call it the angular radius of the ring. Twice this quantity is the angular diameter, denoted by a.

Let P be the integral number designating the order of the ring, while a is the angular diameter. The order of interference at the center, where $i=0$, will be represented by p. Thus we have $p=\frac{2e}{\lambda}$, and consequently $P=p\cos\frac{a}{2}$. The angle being small, this gives

$$p=P+P\frac{a^2}{8}.$$

We shall see later how to determine the integer P; the measurement of a will then permit us to calculate p. For this measurement we use the first or second ring from the center. Let λ' be the wave-length of another radiation. Upon repeating the above determinations, we shall have

$$p'=\frac{2e}{\lambda'}=P'+P'\frac{a'^2}{8},$$

whence,

$$\lambda=\lambda'\frac{p'}{p}.$$

The wave-length λ being that which is to be measured, and λ' that which serves as the fundamental standard, the problem is solved.

We have supposed the integers P and P' to be known. For P' we use the coincidence method described by Fabry and Perot, starting with an approximate determination of the thickness, and employing the radiations of mercury and cadmium. Having, in addition, measured a', we may compute p'.

One has always an approximate value λ_1 of the wave-length λ. The equation $p\lambda=p'\lambda'$ then gives a means of calculating an approximate value p_1 of the order of interference p. If the error of the number λ_1 is not too great, the order of interference of the central ring is the integer next smaller than p_1. Admitting this result, and having measured the diameter a of one of the rings, one obtains the order of interference at the center p. The fractional part of p as measured should not differ from the calculated fractional part of p_1 by more than 0.1 or 0.2, if there is to be no uncertainty as to the integral part.

In a word, the whole matter reduces to the measurement of the angular diameters of the rings produced by the radiations to be compared.

The ray λ being, in our experiments, a ray of the iron arc, it is necessary, in order to observe its interference figures, to separate it from the other rays emitted by the same source. In the experiments of Perot and Fabry, which were made visually, this separation was secured by projecting a spectrum of the source, and isolating by means of a slit, the line to be measured. The light which passed through then fell upon the interference apparatus, and produced the rings. The actual measurements were made entirely by the photographic method. The old method would, under these conditions, have been difficult to apply. We were led to adopt an arrangement such that there would appear on each plate the interference figures produced by a large number of lines. To secure this result it was only necessary to place the interference apparatus in front of the dispersive piece.

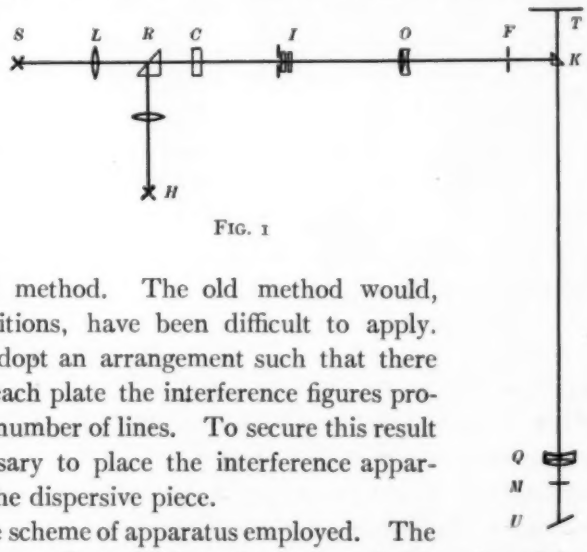


FIG. 1

Fig. 1 shows the scheme of apparatus employed. The light emitted by the arc S passes through a converging lens L, and the interference apparatus I. If necessary, an absorption cell is interposed at C, in order to remove portions of the spectrum which are not wanted. The objective O projects the interference rings upon its focal plane F. According to this plan, each radiation gives its own system of rings, all of which have their centers at the same point. All that remains is then to separate the various radiations. The slit of a spectroscope is placed in the plane F, and so as to coincide with a diameter of the system of rings. The spectroscopic apparatus, provided with a plane grating (it is necessary to avoid astigmatism), thus yields as many images of the slit as there are separate wave-lengths in the incident light. Each of these images

presents the result which would be obtained if the source emitted the corresponding radiation only, upon the portion of the plane F which is occupied by the aperture of the slit. The latter should be of such a width as to prevent the image corresponding to the radiation to be measured from encroaching upon the adjacent images. With spectra having few lines one may employ a very wide slit, thus obtaining complete rings; if the spectrum is very rich in lines the slit must be narrow, and the image then reduces to the bright points in which the slit intersects the rings. Fig. 2 presents both of these cases.

FIG. 2

The arrangement of the dispersive apparatus varies according to circumstances, as explained in subhead IV. That represented in the diagram, Fig. 1, is the auto-collimating system. The image is thrown upon the photographic plate T. It is possible to measure on the same plate the rings due to all the rays present in the photographic portion of the spectrum.

In addition, it is necessary to obtain the rings produced by a comparison line. We have employed for this purpose the green mercury line emitted by the Cooper-Hewitt lamp, placed at H. A totally reflecting prism R, introduced into the incident beam, directs the mercury light, instead of that from the arc, into the interference apparatus. The spectroscope becomes useless for light, which by reason of having been passed through suitable absorption cells may be regarded as monochromatic. A plane mirror, interposed at M, in front of the grating, throws upon the plate T a real image of the rings of the mercury line.

It is necessary to determine the angular diameters of the rings. For this purpose, we measure, upon the plate, the diameter, AB, of one of these, whence we may deduce the angular diameter if we know the scale of the image, that is, the number of millimeters corresponding to a given angle. To accomplish this, there are placed across the slit two metallic landmarks, which project upon each image two straight lines, XY (Fig. 2). When AB is measured there is also made a measurement of the distance $XY=r$, of the images of these reference wires. The angle corresponding to the distance XY has

been previously determined, once for all. Thus we have all the elements necessary for the computation.

IV. DESCRIPTION OF APPARATUS

The optical arrangement which is about to be described requires modification according to the portion of the spectrum to be studied. For the ultra-violet beyond λ 3600 all glass must be eliminated. We shall describe in succession the different pieces of apparatus composing the outfit.

Interferential apparatus.—This consists of two transparent plates, each presenting a plane silvered face. These are kept exactly parallel to each other, and at an invariable distance apart. The apparatus which has been previously described under the name of standard of thickness,[1] was constructed by Jobin. For wave-lengths greater than 3600 the plates are of glass. They were silvered either by the chemical method or by cathode projection, using the apparatus of our colleague Houllevigue.[2] This last method yields silver films of remarkable regularity, and of very high reflecting power, in spite of considerable transparency. Unfortunately, the properties of the silver thus obtained are not always the same, and it is difficult to produce a film which shall meet given conditions.

For wave-lengths below 3600 quartz plates are used. The reflecting power of chemically deposited silver becomes very small in the neighborhood of λ 3300, and the interference figures disappear. Some of the silverings obtained by cathode projection behave in the same way; with others, the reflecting power does not fall off so rapidly, so that we have been able to use them for very short wave-lengths; however, their reflecting power becomes feeble toward the end of the ultra-violet spectrum, and in this region we have been obliged to make use of quartz plates nickeled by cathode projection.

From the standpoint of precision of measurement, it is advantageous to employ interferences having a large difference of path; but there is a limit to this in the fact that the lines are not infinitely narrow, so that the fringes lose their sharpness when the thickness of the air-film becomes too great. The orders of interference used

[1] See footnote 1, p. 171.

[2] *Journal de Physique* (4), 4, 396, 1905.

have varied from 14,000 to 27,000. For wave-lengths greater than 3600 the thickness of the air-film was 5 mm, while for those less than 3600 it was 2.5 mm.

A diaphragm was used in the interference apparatus in such a way as to employ a field only 8 mm in diameter, this being kept the same for all the rays compared. Thanks to this precaution a slight lack of parallelism of the surfaces was without influence.

The apparatus is carried upon a support, constructed by Jobin, which admits of various motions: a double translation allows it to be brought into the beam of light, and it may then be oriented by two rotations in such a way as to center the rings on the slit.

Dispersive apparatus.—Since it is essential to avoid astigmatism, the use of a concave grating is out of the question. We have made use of plane gratings with an arrangement for auto-collimation. Light which has passed the slit is reflected by the totally reflecting prism K (Fig. 1), and falls upon the objective Q, of one meter focal length, which is moved back and forth by a rack and pinion, its position being determined by means of a scale of millimeters engraved upon the sliding tube. The plane grating U is a Rowland grating having a ruled surface of 80 by 50 mm and ruled with 568 lines to the millimeter. It is carried upon a divided circle, by means of which it may easily be placed in a given position. The diffracted light re-enters the objective, passes beneath the totally reflecting prism, and forms the spectrum upon the photographic plate T, of dimensions 9 by 12 cm. This is placed in a metallic holder arranged to slide vertically, in order that several exposures may be made upon the same plate. The distance from the plate-holder to the objective may be changed by a focusing rack; and finally the plate may be set at any given angle with the axis of the objective by rotating it about the vertical line through its center. The whole is supported on a very solid cast framework made by Jobin.

When one wishes to photograph a definite portion of the spectrum, it is essential that one be able, rapidly and without hesitation, to bring each piece—grating, objective, and plate—into the proper position. To this end the following quantities have been determined in advance, and graphically represented as functions of the wave-length: (1) the inclination of the grating, which is easily calculated,

(2) the focusing position of the objective, which is experimentally studied, (3) the inclination to be given the plate in order that the spectrum shall be sharply defined through the greatest possible range, this inclination being readily deducible from the above-mentioned graph and from a study of the curvature of the field. As to the most favorable position of the plate with respect to the luminous ray, this remains invariable, and has been determined once for all.

In the case of wave-lengths greater than 3600, the objective, which is of glass, is achromatic. Its dispersion is very slight, and consequently the inclinations of the plate are small.

For the ultra-violet, the totally reflecting prism K and the objective Q are of quartz. We no longer have achromatism, and the changes of focus are considerable, as are also the inclinations of the plate.

The plane grating which was employed in the visible spectrum showed an unexpected peculiarity. Its spectra of different orders and on both sides ceased abruptly at about λ 3500. Kayser and Runge have observed a grating which exhibited the same anomaly.[1] It does not seem possible to attribute this to the form of the grooves; it is perhaps due to the optical properties of the metal on which the grating is ruled.

The auto-collimating arrangement, while exceedingly convenient, has the slight disadvantage that light reflected by the objective falls upon the plate, and may produce some fog. This is not troublesome except in the case of long exposures. For red radiations, of course, the exposures are long, but the more actinic rays may be removed by an absorption cell, so that the fog need not be feared. This, however, is not the case in the extreme ultra-violet. Here we have been forced to abandon the auto-collimating arrangement, and make use of an apparatus with separate collimator and camera (Fig. 3). The positions of collimator and grating remain unchanged, but the diffracted light falls upon a second objective of quartz, which forms the spectra on the plate T'. The camera is fixed, and the spectra are shifted by turning the grating. Focus is secured by displacing the objective Q and also the plate, while the objective Q' remains fixed.

For the most part, we have used the third-order spectrum. The

[1] *Abhandlungen der K. Akademie der Wissenschaften zu Berlin*, 1888.

scale is then 0.18 mm per Ångström. The second-order spectrum has been used in the red, where it is very bright. It has also been used in the extreme ultra-violet, on account of the overlapping of the spectra.

In order to photograph the interference figures of the green mercury line, there is interposed in front of the grating a plane mirror *M*, carried on a support having three clamp-screws which fit into three sockets, thus permitting it to be replaced quickly in position.

Objective of the rings.—We thus designate the lens *O*, which projects the rings upon the slit. For the spectral region studied, the focal plane of this lens should contain the slit of the spectroscope.

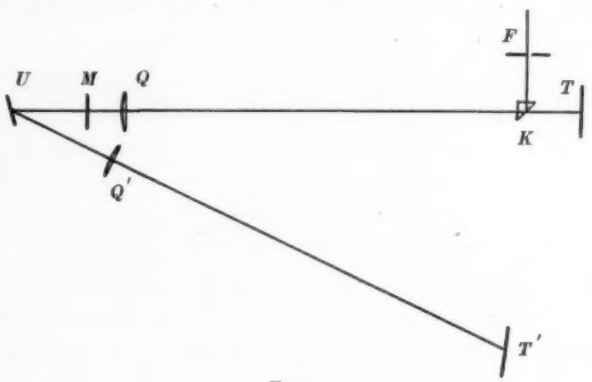


FIG. 3

It is mounted in a graduated sliding tube, and its position may be changed by means of a rack and pinion. It is an achromatic glass lens, replaced in the ultra-violet by a lens of quartz and spar. The focal length of each is about 26 cm. The dispersion of each of these lenses has been studied, and curves plotted, showing the position of the objective for each region of the spectrum.

In each case it is necessary to know the angular distance which corresponds to the distance apart of the standard landmarks placed upon the slit. This angle is defined as follows: suppose the slit to be illuminated from behind; an observer viewing it through the objective of the rings sees it at infinity, together with the marks which it carries. The angular distance of the marks thus viewed is the angle in question. This has been directly determined for the green

mercury line, by using a telescope focused at infinity, and furnished with a micrometer eyepiece which has been previously calibrated in terms of angle by means of a divided circle. The angle θ for any radiation whatever, λ, is derived from the angle θ', measured in the manner just described for the green ray, by means of the formula $\theta = \theta' \frac{f'}{f}$, f and f' being the focal distances determined as already explained.

Absorption cells.—In many cases, it is desirable to eliminate certain troublesome rays. For this we employ cells containing liquids, placed at *C* (Fig. 1). In order to photograph the fringes in the green light of mercury, we interpose a solution of normal chromate of potassium, which eliminates the violet and ultra-violet rays. The plates which we use are not affected by the yellow rays.

When the grating is used, it is necessary to prevent the overlapping of spectra. The chromate cell suppresses all radiations of wave-length less than about 5000. An acid solution of sulphate of quinine absorbs radiations shorter than 4200. On the other hand, it may be necessary in the ultra-violet to eliminate rays of greater wave-length than those which are being studied. This service is rendered by a quartz cell, filled with a solution of nitrosodimethylaniline, as pointed out by Wood.[1] When no absorption cell is required, a cell filled with water is introduced, to diminish the heating of the interference apparatus.

Photographic plates.—One item of great importance is the reduction of the length of exposure. For all the negatives which contain no lines of wave-length less than λ 4800 (the region of sensitiveness of non-orthochromatic plates), we have used the Σ plates of Lumière. In the green region, and for the green mercury line, we have employed Smith plates. In the yellow and red we have made use either of Lumière Orthochromatic *B* plates, or of Cramer Trichromatic. Not until near the end of our research did we try the excellent plates of Wratten and Wainwright, the use of which would have rendered us great service, had we known of them earlier.

V. METHOD OF PROCEDURE

Mounting and adjustment.—The spectroscope, which is the heaviest part of the outfit, remains immovable, and the adjustment

[1] *Astrophysical Journal*, **17**, 133, 1903.

is accomplished by displacing other portions of the apparatus. The sources of light, iron arc, and Hewitt lamp, with its total reflection prism, are first set up. The objective of the rings is centered upon the beam of light. The interference apparatus requires careful mounting and adjustment. Its adjustment consists in securing parallelism between the silvered surfaces. This adjustment is made by turning the clamp screws by which the glass plates are attached to the metal surfaces, and is verified by viewing with the naked eye the rings produced by the mercury light; if the eye be displaced in all directions, the diameter of these rings must remain invariable. The change in thickness of the air-film, which occurs either upon readjustment or as a result of expansion, is extremely small, and does not cause a variation in the order of interference of more than one or two units. Observation of the coincidences of the green and violet rings of mercury fixes, by simple inspection, the order of interference of the central ring.

Having completed this adjustment, the next step is to place in front of the interference apparatus the diaphragm which limits the portion of it which is made use of. The whole is then centered on the beam of light. Its distance from the objective of the rings is such that this objective projects, through the slit, and upon the grating, an image of the opening in the diaphragm. By this arrangement the interference apparatus and the grating are employed in the same manner for all the points of the image and for all radiations.

The interference apparatus should be so oriented that the center of the rings is projected on the middle of the slit.[1] In order to control this, one observes, by means of an eyepiece placed in the position of the photographic plate, the image produced by the green mercury ray.

The slit-width varies, according as the lines are more or less crowded, from 0.15 to 0.30 mm.

The concentration lens, *L*, projects an image of the arc upon the diaphragm of the interference apparatus. This image is so large

[1] This condition is not essential to obtaining correct measurements; it is sufficient that the center of the rings, for both mercury and iron, shall fall at the same distance from the slit. It is, however, desirable, on account of the sharpness of the points, that the slit cut the rings normally, and therefore that it coincide with their diameter.

that the light from the ends of the iron rods can be eliminated. In the case of the ultra-violet this lens is of quartz, so that in adjusting it to its proper position it is necessary to take account of its lack of achromatism.

Making the negative.—The elements of a measurement include the photographing of the interference figures produced by the green mercury ray, and of those of the rays from the iron arc. To get rid of the effect of temperature changes, it is necessary to expose twice upon mercury, once before and again after the exposure corresponding to iron. These three exposures must be made without touching any part whatever of the interference apparatus or the slit. Upon the negatives thus obtained, the images of the standard marks on the slit are not always easily visible if they chance to fall between two bright rings. Consequently, it is advisable to make an exposure upon both the iron and mercury under the same conditions as before except that the interference apparatus has been removed.

The following is then the series of operations carried out:

1. Exposure on the mercury rings. Total reflection prism R in place, as well as the mirror M. The objective of the rings and that of the spectroscope focused for λ 5460. Chromate cell. Photographic plate normal to the pencil of rays.

2. Exposure on the rings of iron. Prism and mirror removed. Objectives focused for the region to be photographed. Plate inclined at the proper angle. Suitable absorption cell.

3. Exposure made on the mercury rings, exactly as before. The interference apparatus is now removed.

4. Exposure on the standard marks, with mercury light, as in operations 1 and 3.

5. Exposure on the standard marks, with light from iron, as in operation 2.

6. It is well to have on the same plate a spectrum of iron made with a narrow slit. Exposure as in operation 5, after having diminished the slit-width. During the exposure on the rings of iron, the temperature and pressure of the surrounding air are noted. According to circumstances, the six exposures are made upon the same plate or upon two different plates, one for iron and the other for mercury.

The time of exposure varies greatly with the thickness of the

silver films and the region of the spectrum. For the mercury rings it is a few seconds: for those of iron it has varied from one second to thirty minutes.

VI. MEASUREMENTS

The measurements consist in the determinations of the diameters of the rings, or more exactly, in the measurement of the distances apart, AB (Fig. 2), of the black points on the negative which represent the intersections of the bright rings with the slit. In addition, one must determine the distance apart of the images of the reference marks. These distances do not exceed ten millimeters, and under favorable circumstances the precision of the measurements reaches a hundredth of a millimeter. We have employed a longitudinal comparator of very simple pattern.

The negative to be measured is placed vertically in a holder upon the carriage of a dividing engine. It is illuminated by transmitted light, and viewed with a micrometer telescope. Upon the same carriage is mounted a slip of glass graduated in tenths of a millimeter, and this is viewed through a micrometer microscope. One observer has his eye at the telescope, and turns the screw of the dividing engine, while the other takes readings at the microscope and records the readings. Each measurement is made twice, the carriage being displaced first in one direction, then in the other.

These measurements give, for each iron line, and for the mercury line, the distance of the standard marks, the diameter of the first ring, and if possible that of the second. Moreover, the angle corresponding to the distance between the standard marks is known (see above). For each radiation the order of interference at the center must be calculated.

Let us consider first the mercury ray. Let r' be the distance between the standard marks, θ' the corresponding angle, expressed in radians, and δ' the diameter of that one of the measured rings the number of whose order, P', is known. The angular diameter of this ring is $a'=\frac{\delta'}{r'}\theta'$. The order of interference at the center is then

$$p'=P'+\frac{P'a'^2}{8}=P'+\frac{P'}{8}\left(\frac{\delta'\theta'}{r'}\right)^2.$$

If we measure the diameters of two rings, we have two values of

p' which ought to be identical: in general they agree to within a few thousandths.

The mean is taken of the results obtained by the two exposures on the mercury ray. By reason of the presence of the observers and of the light sources, the temperature rises somewhat between the two exposures, so that the second of these gives for p' a value a few thousandths higher.

Consider now a ray of iron. Let λ be its wave-length, which is unknown. We know the values of r, δ, θ corresponding to the rings of this ray. The ring whose diameter is δ has for the number of its order the integer P which is not yet known. We proceed to determine this by means of the known approximate value λ_1 of the wave-length λ. This value is taken from Rowland's tables, and is multiplied by 0.999967, the mean ratio between the true wave-lengths and those of Rowland.

The equation $p\lambda = p'\lambda'$, upon substituting for λ its approximate value λ_1, gives an approximate value p_1 of the order of interference, p. If the approximation is sufficiently close, no error is introduced in the integral part. The number of the first ring is the integer next smaller than p_1, and this determines the numbers of all the rings, and in particular the number P of the ring measured. We then calculate the order of interference at the center by the formula

$$p = P + \frac{P}{8}\left(\frac{\delta\theta}{r}\right)^2.$$

An error of a unit in the approximate value need cause no alarm: for example, the order of interference being say 20,000, it would be necessary for the approximate value λ_1 to be in error by 1/20,000, in order to cause a like error. As a matter of fact, the error is much less; the fractional part of p, the order of interference as measured, differs from that of the approximate value p_1 by a quantity considerably less than unity. In general the difference does not exceed 0.1.

The orders of interference are thus determined with an uncertainty of less than 0.01. As they are all in the neighborhood of 20,000, the relative error is less than 1/2,000,000. In the most unfavorable cases, the uncertainty in the value of λ does not reach one part in a million.

For a given negative, the order of interference, p', relative to

mercury, is a constant. We calculate, once for all, the product $p'\lambda'$ which represents twice the thickness of the interference apparatus. This thickness is thus determined with an accuracy of the order of 0.002 μ.

VII. CORRECTIONS

1. *Correction for phase.*—We have proceeded on the assumption that the optical thickness of the air-film is the same for all radiations. This is not rigorously true, on account of the change of phase at reflection from silver, which varies very slightly with wave-length. Everything takes place just as if each radiation suffered, without change of phase, reflection at a certain plane, which may be called the optical surface of the silvered glass, the position of this plane changing slightly from one ray to another.

In order to escape the influence of this variation, it is sufficient to make, at the start, two observations with the same silverings, using two different thicknesses of air. This is, as we shall see, what we have done, but one of the thicknesses used is so small that the exact knowledge of the wave-lengths is useless for calculating the corresponding results, and the observations made with the small thickness solve in themselves the problem of change of phase.

Let λ' and λ be the wave-lengths of the green mercury line, and of any other radiation, respectively. The corresponding thicknesses of the air-film have the very slightly different values e' and e. The question resolves itself into the determination of the difference $e-e'$, as we proceed to show.

The corresponding orders of interference are $p'=\frac{2e'}{\lambda'}$ and $p=\frac{2e}{\lambda}$, whence,

$$\lambda=\frac{e}{e'}\times\frac{p'}{p}\times\lambda' .$$

Instead of making the calculation in this manner, we have reasoned as if the thickness had been the same, while we have given to λ the very slightly inexact value $\frac{p'}{p}\lambda'$. We may correct this value by adding to it the correction

$$\eta=\frac{e}{e'}\times\frac{p'}{p}\times\lambda'-\frac{p'}{p}\lambda'=\frac{2(e-e')}{p} .$$

The probem will be solved if for each radiation we know the difference $\epsilon = 2(e-e')$ of the doubled optical thicknesses corresponding to the two radiations λ and λ'. This difference is evidently independent of the thickness itself; λ' being fixed, it is a simple function of λ, which may be determined, for given surfaces, once for all.

It is sufficient for this purpose to make use of interferences of a low order, using rays whose wave-length is quite closely known. With the same surfaces, let us form an air-film only a few microns in thickness, in such a way that the orders of interference, q and q', are not greater than a score or so. These quantities being once known, we have $2e = q\lambda$ and $2e' = q'\lambda'$, and since q and q' are very small, these equations give e and e' with very great absolute precision, even though the wave-lengths are not very exactly known. We may then calculate the difference $\epsilon = 2(e-e')$. Having repeated this process for various radiations, one may plot a curve giving ϵ as a function of λ. Since, in general, this function does not vary rapidly it is sufficient to make the measurements for a few radiations distributed throughout the spectrum.

Having obtained this curve, it is easy to plot a correction curve applicable to these silverings when employed with a given air-thickness. We have

$$\eta = \frac{\epsilon}{p} = \frac{\epsilon}{2e}\lambda .$$

The problem reduces itself to the observation, for certain radiations, of the orders of interference, while working with a film of very small thickness. We have used thicknesses of the order of $8\,\mu$. We have made use of the fringes of thin plates, as being more convenient, in this case, than the rings at infinity. They are produced by parallel light, and exhibit the lines of equal thickness of the air-film.

Upon one of the silvered surfaces there have been traced with a fine point several standard marks arranged along a straight line, and each consisting of two intersecting straight lines. The two silvered surfaces are then superposed, being held apart by three strips of thin tinfoil placed around their edges. The whole is then tightly held in a screw clamp. Viewing this by transmitted mono-

chromatic light, one sees rectilinear fringes, whose distance apart and orientation may be varied by turning the screws of the clamp. They are thus brought into perpendicularity to the line of standards. We first examine them by mercury light. To each point of the film, and in particular to each standard, corresponds, with the green light, a definite order of interference. The numbers of the green fringes are easily obtained by examining their coincidences with the yellow and violet fringes. The order of interference corresponding to any standard is deduced from a measurement of the distances of this standard from the neighboring fringes. In Fig. 4 the fringes are indicated by black lines, and their numbers are shown. For the standard R the value of the order of interference is $35+\frac{RA}{AB}$. Let q' be the order of interference thus found. This process is repeated for any other radiation, λ; and the order of interference q determined in the same way: its fractional part by interpolation, and its integral part by calculating in the first place the approximate value of q, namely, $\frac{q'\lambda'}{\lambda}$. One may then calculate the values of the double optical thicknesses $2e=q\lambda$ and $2e'=q'\lambda'$ and consequently $\epsilon=2e-2e'$.

We have employed most frequently the radiations of mercury (the Hewitt lamp for the visible spectrum, the quartz lamp of Heraeus for the ultra-violet). In certain cases we have used the rays of the iron arc. Nearly all the measurements have been made photographically. An image of the thin film is projected upon the slit of the spectroscope in such a way that the image of the standards traced on the film shall fall on the slit. The measurement of the negatives is made with the help of the longitudinal comparator previously described. Settings are made on the several fringes and on the standards, and from these are deduced the order of interference of each. These quantities are thus determined with an uncertainty of less than 0.01, that is to say, of the same order as is attained in the measurement of the rings at infinity.

In this computation the wave-lengths are regarded as known, but it is not necessary to know them with great precision. Supposing that we have fringes of order 40, q is determined to 1/4000, very nearly. If λ is in error by 0.1 Ångström, this will cause in $2e$ an

error ten times less than that which results from the uncertainty of q, and therefore negligible.

These experiments have been repeated for each of the silverings which we have used. The properties of silverings chemically produced seem to be always identical among themselves, and depend only on the thickness. For these we have obtained results in the same sense and of the same order of magnitude as those indicated by Perot and Fabry. As one approaches wave-length 3300, not only does the reflecting power rapidly diminish, but the change of phase also undergoes a rapid variation. On the contrary, the silverings obtained by cathode projection do not all behave alike as regards the size, or even the sense, of the difference ϵ.

2. *Correction for the temperature and pressure of the air.*—It has been decided that all wave-lengths shall be expressed in terms of their values in dry air at 15°, and under normal pressure, and the number adopted for the wave-length of the red ray of cadmium was obtained under these conditions. Variations in the condition of the air cause the wave-lengths to change in absolute value by quite considerable amounts. On the other hand, the ratios of wave-lengths vary by very small quantities only, for the effect is due entirely to the dispersion of the air. If the index of refraction of the air were constant through the spectrum, all wave-lengths would vary in the same ratio. If this were the case, our measurements would give the wave-lengths directly as under normal conditions, whatever might be the state of the atmosphere at the time of making the measurements, since the value adopted for the fundamental wave-length and introduced into the calculations is that corresponding to these normal conditions. However, the dispersion of the air is not zero; hence a correction must be made, although very slight, depending upon the atmospheric conditions at the time of obtaining the negative. By "actual conditions" we shall designate those which exist at this moment, and by "normal conditions" those corresponding to 15° and normal pressure.

Let M be the specific mass of the air under actual conditions, defined by the temperature t and the pressure H; λ the wave-length of a ray under actual conditions, n the corresponding index of the air under the same conditions; λ' and n' the analogous quantities

for the green ray of mercury. The same letters with subscript zero will represent the same quantities under normal conditions. The quantity which we wish to find is λ_0. We have measured the orders of interference p and p' under actual conditions, and hence obtained $p\lambda = p'\lambda'$. The calculation was conducted in the following manner: we have computed the quantity $\frac{p'\lambda'_0}{p}$, which would be the normal wave-length if the air were not dispersive, and to this has been applied a correction.

This correction, which is the difference between the exact value and that which is to be computed, has the value

$$\gamma = \lambda_0 - \frac{p'\lambda'_0}{p} = \lambda_0\left(1 - \frac{p'}{p} \times \frac{\lambda'_0}{\lambda_0}\right) = \lambda_0\left(1 - \frac{\lambda\lambda'_0}{\lambda'\lambda_0}\right),$$

but

$$\frac{\lambda}{\lambda_0} = \frac{n_0}{n} \text{ and } \frac{\lambda'}{\lambda'_0} = \frac{n'_0}{n'},$$

and hence

$$\gamma = \lambda_0\left(1 - \frac{n_0\, n'}{n'_0\, n}\right) = \lambda_0 \frac{n'_0\, n - n_0\, n'}{n'_0\, n},$$

in which the denominator $n'_0\, n$ may be replaced by unity. We have also,

$$\frac{n-1}{M} = \frac{n_0 - 1}{M_0} \text{ and } \frac{n'-1}{M} = \frac{n'_0 - 1}{M_0}.$$

Solving these equations for n and n', and introducing these into the expression for γ, this becomes

$$\gamma = \lambda_0(n_0 - n'_0)\frac{M - M_0}{M_0}.$$

The factor $\frac{M - M_0}{M_0}$, which is easy to calculate as a function of the temperature and pressure, is constant for all the rays of any one negative. Moreover, the quantity $\lambda_0(n_0 - n'_0)$ has been calculated as a function of the wave-length, and plotted in a curve.

These corrections do not reach a value of any consequence, except in the extreme ultra-violet; the largest we have met with is about 0.007 Ångström.

3. *Measurement of the green mercury ray.*—The green mercury line is accompanied by numerous satellites; hence it is unsuitable for a primary standard. We have used it merely as an intermediary,

on account of its brightness and the convenience attending its employment. Its wave-length has been compared with those of the cadmium lines, and this for every thickness of air-film employed, and for each of the silverings. In this way has been eliminated all error due to the influence of the satellites. This comparison was made visually, and without dispersive apparatus. One may transmit through the interference apparatus light from either source at will, suitable absorption-cells being interposed. The rings are observed by means of a telescope focused at infinity, and provided with a micrometer which has been calibrated for angular readings.

VIII. RESULTS

The number of negatives used in these measurements was 43, of which 22 were obtained with glass apparatus, and 21 with quartz. Each plate contains, usually, an interval of 600 Ångströms, but in the regions in which the defects of achromatism make themselves chiefly felt, the whole extent of the plate is not available. In the most favorable cases, we have measured fifteen or twenty lines on the same plate. The same part of the spectrum always occurs on several plates.

The total number of measurements carried out has been 405, involving 115 different lines.

We give here a complete example of a measurement.

Conditions under Which the Negative Was Made

Negative 56. March 17, 1906. Smith plate. Temperature 17°.6. Pressure 765.

Interference apparatus of 5 mm thickness. Glass plates silvered by cathode projection.

Slit-width 0.3 mm.

For the mercury rings, time of exposure 30 seconds.

For the iron rings, time of exposure 10 seconds, wave-length at middle of plate 3800. Inclination of the plate 12°. Intensity of arc current 3 amperes.

Measurement of the Mercury Rings

	1st Ring $P'=18336$	2d Ring $P'=18335$	Distance of Standards
Exposure 1	$\delta'=2.675$	$\delta'=6.11$	$r'=8.12$
Exposure 2.....	$\delta'=2.735$	$\delta'=6.115$	$r'=8.12$

All these lengths are expressed in millimeters.

Angular distance of the standards placed upon the slit, $\theta' = 0.03091$.

The order of interference is then calculated from the formula

$$p' = P' + \frac{P'}{8}\left(\frac{\theta'\delta'}{r'}\right)^2,$$

which gives the following values:

	Calculated from 1st Ring	Calculated from 2d Ring	Mean
Exposure 1	18336.238	18336.242	18336.240
Exposure 2	.248	.244	.246

Final mean, $p' = 18336.243$

The wave-length of the green ray, obtained with these silverings and this interference apparatus, is $\lambda' = 5460.741$.

Consequently $p'\lambda' = 10012.947\ \mu$.

Measurement of the Iron Rings

Wave length given by Rowland		3977.891
The same corrected		$\lambda_1 = 3977.76$
Approximate value of the order of interference		$p_1 = \frac{p'\lambda'}{\lambda_1} = 25172.32$
Angle between standards		$\theta = 0.03040$ radian
Distance between standards		$r = 8.09$ mm
	1st Ring	2d Ring
Diameter of rings, δ	3.02	5.625
Number of order, P	25172	25171
Computed order of interference	25172.406	25172.407
Mean, p	25172.406	
Rough value of wave-length, $\frac{p'\lambda'}{p}$		3977.7473
Correction for air		0
Correction for phase		−0.0019
Corrected wave-length		3977.7454

All the lines have been measured several times; some of them with the surfaces of the interference apparatus covered with different metals (silver deposited by cathode projection, chemically deposited silver, and nickel). The results, before being corrected for phase, are extremely discordant; after these corrections are applied they become very concordant. As an example of this, we may give the numbers obtained with a line situated in the region of the spectrum in which the properties of chemically deposited silver vary rapidly,

and where, consequently, the corresponding phase corrections are considerable. In the following table, we give the numbers of the negatives, the nature of the metallic surface, the value of the wave-length corrected for the effect of air, but not for phase-change, the value of this correction, and finally the corrected wave-length λ. The integral part of the wave-length, 3399, is not repeated in the table.

	Silver Deposited from Cathode		Silver Deposited Chemically				Nickel
Negative	134	136	175	177	194	195	254
λ uncorrected	3399.3440	.3440	.3140	.3145	.3128	.3134	.3280
Correction for phase	−.0064	−64	+233	+233	+233	+233	+91
λ corrected	3399.3376	.3376	.3373	.3378	.3361	.3367	.3371
Mean				3399.3372			

The following table gives the definitive result of the measurements:

2373.737	3556.879	4647.437	5535.418
2413.310	3606.681	4678.855	5569.632
Si 2435.159	3640.391	4707.287	5586.770
Si 2506.904	3677.628	4736.785	5615.658
Si 2528.516	3724.379	*Mn* 4754.046	5658.835
2562.541	3753.615	4789.657	5709.396
2588.016	3805.346	*Mn* 4823.521	*Ni* 5760.843
2628.296	3843.261	4859.756	5763.013
2679.065	3865.526	4878.226	*Ni* 5805.211
2714.419	3906.481	4903.324	*Ni* 5857.760
2739.550	3935.818	4919.006	*Ni* 5892.882
2778.225	3977.745	4966.104	5934.683
2813.290	4021.872	5001.880	5952.739
2851.800	4076.641	5012.072	6003.039
2874.176	4118.552	5049.827	6027.059
2912.157	4134.685	5083.343	6065.493
2941.347	4147.677	5110.415	6137.700
2987.293	4191.441	5127.364	6191.569
3030.152	4233.615	5167.492	6230.732
3075.725	4282.407	5192.362	6265.147
3125.661	4315.089	5232.958	6318.029
3175.447	4352.741	5266.568	6335.343
3225.790	4375.935	5302.316	6393.612
3271.003	4427.314	5324.196	6430.859
3323.739	4466.554	5371.498	6494.994
3370.789	4494.572	5405.780	
3399.337	4531.155	5434.530	
3445.155	4547.854	5455.616	
3485.344	4592.658	5497.521	
3513.820	4602.944	5506.783	

Comparison with previous results.—Among the lines which we have measured are the fourteen lines visually measured by Fabry

and Perot in 1901. These older measurements were founded upon the value 6438.4722 for the red cadmium line, as given by Michelson and Benoit. Upon reducing these to the same unit as the actual measurements, they become very concordant with these latter. The differences amount to but a few thousandths of an Ångström, sometimes in one sense, sometimes in the other. One difference only is as great as 0.007. The mean of the differences, taken with their proper signs, is only 0.0006; there is therefore no systematic variation between the two series of determinations. The mean of the differences taken without regard to sign is 0.004, which is less than one-millionth in relative value.

A comparison of our numbers with those of Rowland would have no definite significance, because the one set is based on the spectrum of iron, while the other is founded on the solar spectrum, and there is no identity between the two sorts of lines. If the ratios of Rowland's numbers to ours be taken, they are always found to vary systematically through the spectrum in such a way as to confirm the results announced by Perot and Fabry.

A REDETERMINATION OF THE WAVE-LENGTHS OF STANDARD IRON LINES

By A. H. PFUND

As it has been shown by Fabry and Perot[1] and by Kayser[2] that the grating method is not well adapted for the accurate determination of secondary standards of wave-length, it has been decided by the International Union for Co-operation in Solar Research that the problem be attacked by means of the interference method of Fabry and Perot. In order to establish these new standards with definiteness, determinations have been carried out by the same method and on the same iron lines by Fabry and Buisson[3] at Marseilles, by Eversheim[4] at the Physical Institute of the Universität Bonn, and by the writer at the Physical Laboratory of the Johns Hopkins University.

GENERAL DESCRIPTION OF APPARATUS

As the theory of the Fabry and Perot interferometer and the formulae used in the calculation of wave-lengths have been fully given by Fabry and Perot,[5] Lord Rayleigh,[6] Eversheim,[7] and Zeeman,[8] it would be unnecessary repetition to discuss these matters once more. I shall therefore limit myself to a description of the apparatus and a discussion of applications of the formulae. The apparatus may be described briefly as follows: A mirror at M (Fig. 1), consisting of two thin pieces of plane glass, was so adjusted as to give the same direction to the two beams of light coming, respectively, from the cadmium lamp (C) and the iron arc (I). After having been

[1] Fabry and Perot, *Astrophysical Journal*, **15**, 261, 1902.

[2] Kayser, *ibid.*, **19**, 157, 1904.

[3] Fabry and Buisson, *Comptes Rendus*, **143**, 165, 1906, and **144**, 1155, 1907.

[4] Eversheim, *Astrophysical Journal*, **26**, 172, 1907; *Zeitschrift für Wiss. Photographie*, **5**, 152, 1907.

[5] Fabry and Perot, *Astrophysical Journal*, **15**, 73, 1902.

[6] Lord Rayleigh, *Phil. Mag.*, **11**, 685, 1906.

[7] Eversheim, *loc. cit.*

[8] Zeeman, *Physikalische Zeitschrift*, **7**, 209, 1908.

made convergent by means of the lens (L), the light is reflected downward by means of the totally reflecting quartz prism (T_1), passes through the interferometer plates (P, Fig. 2), and is once more given a horizontal direction by the quartz prism (T_2). It is next reflected from the plane speculum mirror at N (Fig. 1) to the concave speculum mirror at O which in turn reflects the beam almost at normal incidence and projects it upon the slit (S). As the slit is placed in

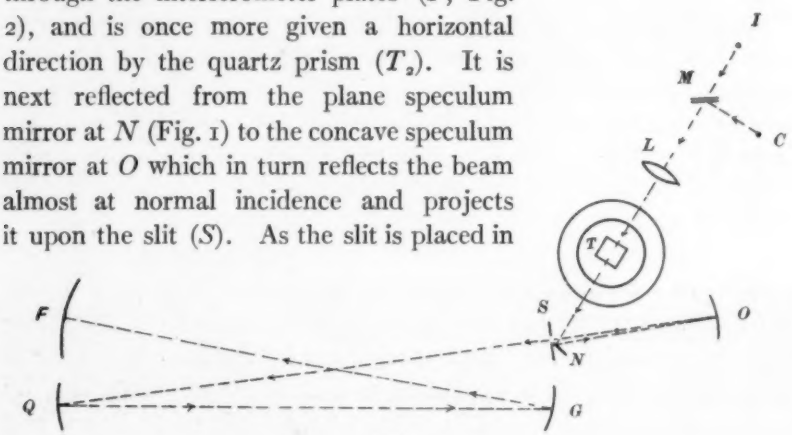


FIG. 1

the principal focus of the concave mirror, and as the latter does not suffer from the defects introduced by chromatic aberration, it is evident that the entire fringe system is brought to a focus simultaneously upon the slit. As the quantity made use of in the final calculation of wave-length is the angular diameter of a fringe, the fringe system upon the slit is so adjusted that the slit lies along a diameter of the rings. By means of a concave mirror at Q a beam of parallel light is made to fall upon the concave grating at G which finally produces a spectrum at F. By using a concave mirror in conjunction with the concave grating it is evident that the spectrum at F consists of monochromatic, non-astigmatic slit images—each image showing its own fringe system. The appearance of photographs as obtained with this arrangement of apparatus is shown in Fig. 3, where the lower reproduction is due to the green and blue lines

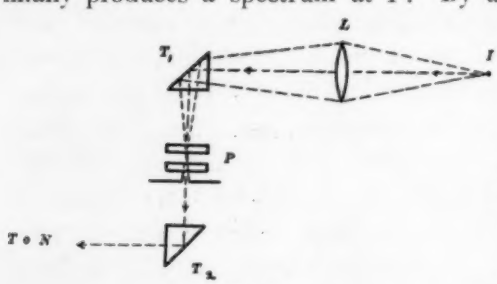


FIG. 2

of cadmium taken with a very wide slit, and the upper, to iron (ratio of enlargement 2.5:1).

The mirror at *O* had a focal length of 20 cm, while the mirror (*Q*) as well as the concave grating (*G*) had a focal length of about 1 meter. In order to bring all of the lines to a sharp focus the photographic film was bent along an arc having a radius of curvature of 50 cm, and in order to avoid astigmatism as much as possible, the grating was so adjusted that a normal drawn from the center of the grating

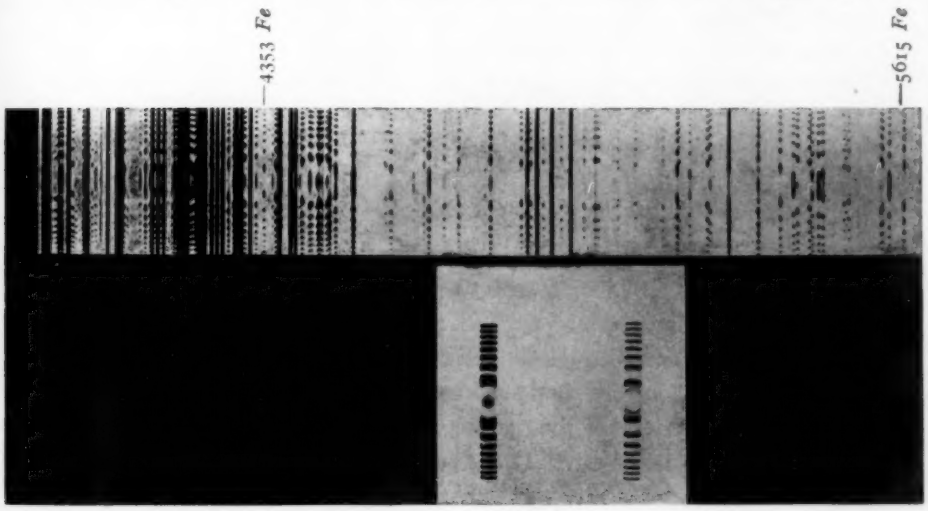


FIG. 3

passed through the middle of the photographic film. Furthermore, the length of spectrum photographed was limited to 5 cm on either side of the normal. The range of wave-length recorded on a film 10 cm in length extended, in the first-order spectrum, from 6500 Å. U. to 3500 Å. U. In view of some proposed alterations in the apparatus, the second-order ultra-violet, which practically began at the same wave-length at which the first order ended, was not photographed. In photographing the first-order spectrum a glass lens was placed at *L* and a Seed "ortho" film was used. In view of the fact that these films are not sensitive in the region of the red cadmium line, a small piece of Wratten and Wainwright "panchromatic"

plate less than 2 cm in width was used to photograph this part of the spectrum.

MOUNTING OF PLATES; CONSTANT-TEMPERATURE BATH

One of the greatest sources of error encountered in the earlier stages of the present investigation was the change in the diameter of the fringe system due to expansion of the "etalon" separating the plates. Using an "etalon" or ring of brass and mounting the plates without any particular precautions against temperature changes, it was found that during an interval of fifteen minutes—the time usually required for a full exposure—the diameter of the fringe system often changed perceptibly and caused a broadening of the photographic record. In order to eliminate once and for all the uncertainties due to temperature changes, the rings were made of invar and the entire system of interferometer plates and invar ring was placed in a constant-temperature bath. That this heroic procedure was effective is shown by the fact that the variations in the diameter of fringes photographed at an interval of more than six hours were too small to be detected on the dividing engine.

The usual type of etalon was used, consisting of a metallic ring which supported three invar studs, equally spaced around the circumference of the ring. The two etalons constructed gave the interferometer plates a separation of 2.654 and 4.427 mm, respectively. The adjustment of the three studs of any one ring to the same length is a comparatively simple matter. The first step consists in filing these studs down so that their lengths, as measured with a micrometer caliper, reading to hundredths of a millimeter, appear the same. The etalon is next placed between the two half-silvered interferometer plates and the transmitted fringe system due to the light coming from a sodium flame is examined. At first the fringes will not be completely circular but will have the appearance of crescents. In order to remedy this defect, the stud lying on the concave side of the crescent is shortened by being rubbed lightly against a smooth piece of wood or paper. The final test to be applied to this adjustment is that the fringes retain the same diameter no matter through what portion of the plates they are viewed. A little practice is sufficient to enable anyone to make the three studs of so

nearly the same length that the difference between the longest and the shortest is less than one-quarter of the wave-length of sodium light.

The constant-temperature bath in which the plates were mounted consists of an outer galvanized iron portion (*A*, Fig. 4) which is

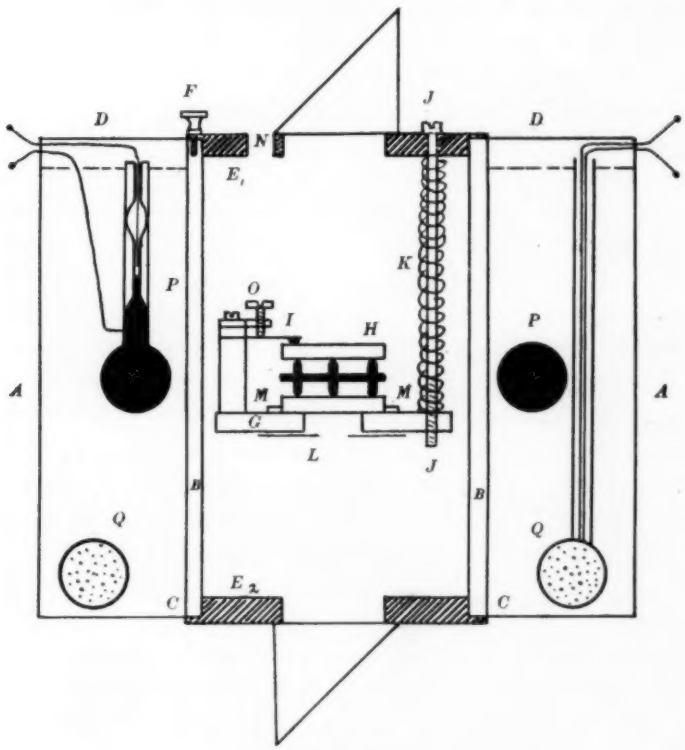


FIG. 4

soldered to a heavy brass tube (*B*) along the edge (*C*). An annular plate of galvanized iron (*D*) covers the top of the tank proper, while two circular plates (E_1) and (E_2) close the ends of the brass tube. The upper brass plate, which is held rigidly in position by two thumb-screws such as *F*, carries the entire interferometer system by means of three brass rods (one of which is shown at *J*). In order to be able to tip the table (*G*) carrying the interferometer plates so as to give the fringes their proper position, the brass rods are threaded at

their lower ends. Rigidity of this portion of the mounting is attained by the use of coil springs (K) having a tendency to expand lengthwise. As shown in the diagram, the quartz plates (H) are adjusted to parallelism by the well-known arrangement (I) due to Fabry and Perot, and are held in position by the brass strips (M, M). These brass strips together with a diaphragm at L containing a circular opening 4 mm in diameter make it possible to return the plates always to the same position and thus insure using the same portion of the silver film in all cases. Final adjustments for parallelism are made by inserting a small screw-driver through an opening (N) and engaging the screw-head (O).

The temperature of the plates was kept constant by means of a so-called "electric thermometer"—a device too well known to require description. The outer portion of the tank was filled with water, as it was found that the vapors of kerosene attacked the silver film on the interferometer plates. The temperature-regulating device (P) consisted of a glass tube bent into the form of a circle and filled with mercury. A similar glass tube (Q) containing oil and numerous strands of German silver wire supplied the necessary heat to the water. By using such a temperature regulator, the temperature of the water (25°0 C.) was kept so nearly constant that the diameters of the fringes did not change perceptibly for hours.

THE SILVER FILM

The two interferometer plates were silvered by means of the well-known Rochelle-salt process, the thickness of silver being such that ten or eleven images could be seen by viewing an ordinary gas-flame through the two mirrors. It was found that these images rapidly assumed a deep, ruby-red color, showing that the number of reflections for red light was much greater than for blue. This is due, not only to the smaller reflecting power of silver for blue light, but also, and to a very marked degree, to a thin, pale-yellow surface film which is always present when silver is deposited chemically. When it is remembered that such a yellow film absorbs blue light very strongly and that the light escaping to form the tenth image has passed through such a film 38 times, it is evident that the removal of this film becomes highly desirable. Heretofore it has been found

impossible to polish semi-transparent silver films, for the reason that the silver is scratched or even rubbed off the glass as soon as it is touched. I have succeeded, however, in overcoming these difficulties by proceeding in the following manner:

After silvering the plates, they are placed in an air-bath (at about 45° C.) for an hour and are then polished by means of a toilet "powder-puff," consisting of innumerable fine feathers. After applying "optical" rouge to the puff, its ivory handle is gripped in the chuck of a drill-press or lathe and the entire arrangement is used as a buffing wheel. By proceeding in this manner very thin silver films may be highly polished without scratching. The very gratifying consequence of subjecting a pair of mirrors to a treatment such as this is that the number of reflections is about doubled. The mirrors used in the work here presented showed ten images before and twenty-one after polishing.

SOURCES OF LIGHT

The standard cadmium lines were produced in a Heraeus fused quartz lamp which was continually connected with a mercury air-pump. By running the lamp on as low a current as 3.5 amperes, supplied by a 220-volt direct-current circuit, the sharpness of the red and green lines was splendid. The iron lines were produced by an arc playing between two iron electrodes and also consuming 3.5 amperes supplied by a 220-volt direct-current circuit. The particular type of arc used, which burns with great steadiness and produces lines but little inferior to those of cadmium, has already been described in an earlier publication.[1] In the final calculations of wave-lengths, Michelson's value for the red and green cadmium lines were used.

CALCULATION OF RESULTS

The formula used in the calculation of wave-lengths was that first given by Fabry and Perot. It takes the form

$$\lambda=\lambda_s \frac{P_s}{P}\left(1+\frac{x^2_s}{8}-\frac{x^2}{8}\right) \qquad (1)$$

where

λ=unknown wave-length of an iron line

λ_s=wave-length of green *Cd* line (5085.824 Å. U.)

[1] *Astrophysical Journal*, **27**, 296, 1908.

P=order of inner fringe of λ
P_s=order o inner fringe of λ_s
x=angular diameter of inner fringe of λ
x_s=angular diameter of inner fringe of λ_s.

As the angular diameter of an inner fringe is small, it is permissible to replace the angle by its tangent, and therefore, if we represent the linear diameter of a fringe by d and the focal length of the concave mirror which projects the entire fringe system on the slit of the spectroscope by R, we have

$$x=\tan x=\frac{d}{R},$$

whence

$$\lambda=\lambda_s\frac{P_s}{P}\left(1+\frac{d_s^2}{8R^2}-\frac{d^2}{8R^2}\right). \qquad (2)$$

As the concave grating (G, Fig. 1) and the concave mirror (Q) were not of identically the same focal length, a slight reduction in the size of the slit image occurred in the focal plane of the concave grating and therefore the diameter of the fringes was less on the photographs than on the slit itself. In order to determine the necessary factor of reduction, two fine parallel lines about 1 cm apart were etched on a piece of glass which was then placed in front of the slit in such a way that the etched lines were in contact with, and ran perpendicular to, the jaws of the slit. Upon photographing the iron spectrum it was found that two very narrow lines, perpendicular to the spectral lines, crossed the entire spectrum. The reduction factor was found by determining the ratio of the separation of these two lines to the separation of the lines etched on the glass plate. In order to facilitate matters in the numerical calculations, d was measured in turns of the screw of the dividing engine and R was reduced to the same units and was multiplied by the above-mentioned reduction factor. Measurements taken over the entire plate showed no perceptible variations in the separation of the two lines, therefore the same value of R applies to all measurements.

For each spectral line the diameters of only the two central fringes were measured as it was found that the outer fringes contributed nothing to the accuracy of the determination. Instead of making a separate wave-length determination for each fringe, the diameter

of the inner fringe was calculated from a knowledge of the diameter of the second, according to the relation:

$$d_1^2 = d_2^2 - \frac{8R^2}{P_2} \tag{3}$$

where the subscripts 1 and 2 refer, respectively, to the inner and second fringe. The two values were then averaged and the wave-length calculation was carried out.

ELIMINATION OF PHASE-CHANGE

The true nature of the phase-change which occurs when light is reflected from a metal is not understood and it is only for the sake of convenience that it is looked upon as being due to a penetration of the light into the metallic surface. Placing, then, this interpretation upon the phenomenon of phase-change, it is evident that the true perpendicular distance between the planes at which the light reverses its direction is greater than the separation of the geometrical surfaces of the silver films. Furthermore, as silver is more transparent to blue light than to red, the amount of penetration and hence the separation of the plates will vary throughout the entire spectrum. Instead of actually determining this penetration for any given wave-length it has been found more convenient to calculate the desired correction factor in terms of a small quantity (ϵ) which must be added to or subtracted from the order of the fringe (P) in order to make the penetration the same as that experienced by the standard green *Cd* line—for which ϵ is arbitrarily placed equal to zero. If the wave-lengths throughout the entire spectrum were accurately known, it would be possible to determine ϵ from the measurements obtained with but one etalon. If, for example, we place ϵ_g equal to zero for the green *Cd* line (λ_g), the value of ϵ_r for the red *Cd* line (λ_r) is given by the relation:

$$P_r + \epsilon_r = P_g \frac{\lambda_g}{\lambda_r}\left(1 + \frac{d_g^2}{8R^2} - \frac{d_r^2}{8R^2}\right),$$

whence

$$\epsilon_r = P_g \frac{\lambda_g}{\lambda_r}\left(1 + \frac{d_g^2}{8R^2} - \frac{d_r^2}{8R^2}\right) - P_r . \tag{4}$$

Unfortunately, however, we are sure of the wave-lengths of only the red and green lines of cadmium, and therefore another procedure must be adopted to obtain the values of ϵ throughout the remainder

of the spectrum where no accurate wave-length determinations exist. As the amount of penetration, and hence the value of ϵ, does not depend upon the separation of the plates, the effect of penetration may be eliminated, as is shown below, by obtaining measurements for two different separations of the plates. After having evaluated the unknown wave-length (λ), ϵ may be found from either of the two equations (5*a*) or (5*b*). In the following, P and d, and P' and d' refer, respectively, to the larger and the smaller separation of the plates, and λ_s refers to the green cadmium line.

$$P+\epsilon=P_s\frac{\lambda_s}{\lambda}\left(1+\frac{d_s^2}{8R^2}-\frac{d^2}{8R^2}\right) \tag{5a}$$

$$P'+\epsilon=P'_s\frac{\lambda_s}{\lambda}\left(1+\frac{d'^2_s}{8R^2}-\frac{d'^2}{8R^2}\right) \tag{5b}$$

subtracting and solving for λ:

$$\lambda=\lambda_s\frac{P_s\left(1+\frac{d_s^2}{8R^2}-\frac{d^2}{8R^2}\right)-P'_s\left(1+\frac{d'^2_s}{8R^2}-\frac{d'^2}{8R^2}\right)}{P-P'}. \tag{6}$$

As the wave-length of the red cadmium line is known, the phase-change for that wave-length was calculated from expression (4). On the other hand, the phase-change for the shortest iron line measured ($\lambda=4282$) was determined from expression (5). These measurements will be given in full later on. Instead of going through the laborious procedure of calculating the phase-change for each line measured, a curve was drawn representing the variations of ϵ with λ and the corrections obtained from this curve were subsequently applied to the measurements given by the greater separation of the plates.

Unless the dispersion of air is taken into consideration no correction need be applied to the measurements as obtained in order to reduce them to standard conditions of temperature and pressure. Calculations show that, for the region of spectrum here investigated, the corrections for the dispersion of air lie entirely below the error of observation and therefore need not be taken into consideration.

NUMERICAL RESULTS

The measurements here given represent an average of a number of complete series of observations. As has been mentioned before,

the measurements used in the wave-length calculations of the iron lines were from the photograph taken with the greatest separation of plates, for in this case the fringes were much sharper and the order of fringe was higher.

PHASE-CHANGE

Thin etalon: $2t=5.3084081$ mm. $\quad \frac{1}{8R^2}=0.0000041867$

$\lambda=5085.824$ (*Cd*) $\quad$ $P_g=10{,}437$

$d_1=3.878$ $\quad d_1^2=15.04$ $\quad d_1^2=15.04$ } 15.02

$d_2=6.155$ $\quad d_2^2=37.88$ $\quad d_1^2=15.00$ }

$\frac{d_1^2}{8R^2}=0.00006288$

$\lambda=6438.4722$ (*Cd*) $\quad$ $P_r=8{,}244$

$d_1=4.859$ $\quad d_1^2=23.61$ $\quad d_1^2=23.61$ } 23.65

$d_2=7.257$ $\quad d_2^2=52.67$ $\quad d_1^2=23.69$ }

$\frac{d_1^2}{8R^2}=0.00009902$

Calculating ϵ_r from expression (4) we find: $\epsilon_r=+0.009$.

PHASE-CHANGE

Thick etalon: $2t=8.8539049$ mm. $\quad \frac{1}{8R^2}=0.0000041867$

$\lambda=5085.824$ (*Cd*) $\quad$ $P_g=17{,}408$

$d_1=3.687$ $\quad d_1^2=13.59$ $\quad d_1^2=13.59$ } 13.61

$d_2=5.230$ $\quad d_2^2=27.35$ $\quad d_1^2=13.63$ }

$\frac{d_1^2}{8R^2}=0.00005698$

$\lambda=6438.4722$ (*Cd*) $\quad$ $P_r=13{,}751$

$d_1=3.100$ $\quad d_1^2=9.61$ $\quad d_1^2=9.61$ } 9.62

$d_2=5.196$ $\quad d_2^2=27.00$ $\quad d_1^2=9.63$ }

$\frac{d_1^2}{8R^2}=0.00004028$

Calculating ϵ_r from expression (4) we find $\epsilon_r=+0.011$.

Hence we find the average value of the two determinations:

$$\begin{array}{r} 0.009 \\ 0.011 \\ \hline +0.010 \end{array}$$

$\therefore \epsilon_r=+0.010$ for $\lambda=6438.4722$

PHASE-CHANGE

Thick etalon: $2t=8.8539049$ mm. $\quad \frac{1}{8R^2}=0.0000041867$

$\lambda=4282.411$ *Fe* (according to F. & B.) $\quad$ **P=20,674**

$d_1=3.507$	$d_1^2=12.30$	$d_1^2=12.30$	12.34
$d_2=4.893$	$d_2^2=23.94$	$d_1^2=12.38$	

$$\frac{d_1^2}{8R^2}=0.00005166$$

Thin etalon: $2t=5.3084081$ mm. $\quad \frac{1}{8R^2}=0.0000041867$

$\lambda=4282.411$ *Fe* (according to F. and B.) $\quad$ **P=12,395**

$d_1=4.071$	$d_1^2=16.57$	$d_1^2=16.57$	16.53
$d_2=5.981$	$d_2^2=35.77$	$d_1^2=16.49$	

$$\frac{d_1^2}{8R^2}=0.00006921$$

Substituting these values in expression (6) we find

$$\lambda=4282.4117.$$

From expressions (5*a*) or (5*b*) we finally obtain

$$\epsilon=-0.020 \text{ for } \lambda=4282.4117.$$

The variations of phase-change with λ are represented by the continuous curve, Fig. 5; the dotted curve is taken from the work of

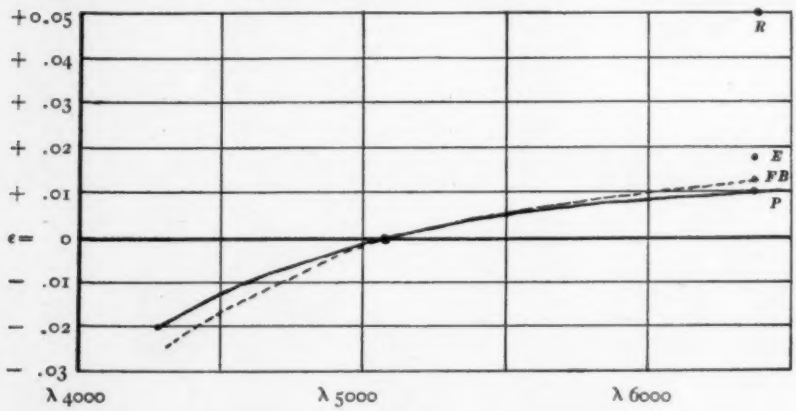


FIG. 5

Fabry and Buisson and the points marked (*R*) and (*E*) are taken, respectively, from the papers of Lord Rayleigh and Eversheim. Although, in all of these cases, the silver was deposited chemically,

the values of ϵ for the same wave-length vary enormously, being dependent, as they are, upon the thickness of layer and condition of the surface of the silver film. As it is not possible to reproduce silver films with perfect exactness, it is necessary to determine the values of ϵ for each set of silver films.

The following are the wave-lengths which I have determined, together with those of other observers who have worked on the same iron lines:

I Pfund (1908)	II Fabry & Buisson (1906)	III Fabry & Buisson (1908)	IV Eversheim (1907)
4282.412	.411	.407	.413
4315.094	.093	.089	....
4352.744	.745	.741	....
4427.316	.318	.314	....
4494.574	.576	.572	.581
4531.156	.159	.155	....
4859.757	.759	.756	.761
4878.225	.229	.226	....
4903.327	.327	.324	....
4919.007	.009	.006	....
5001.885	.883	.880	....
5167.495	.495	.492	....
5232.958	.960	.958	.963
5434.531	.532	.530	....
5455.617	.618	.616	....
5497.523	.523	.521	....
5506.784	.785	.783	....
5586.772	.772	.770	....
5615.659	.660	.658	....
6191.568	.570	.569	....
6230.731	.733	.732	....
6393.611	.612	.613	....
6494.992	.994	.994	....

To show how the values of the wave-lengths were obtained, a typical calculation is given in full.

$\lambda = 5506.785$ *Fe* (according to F. and B.)

Thick etalon: $2t = 8.8539049$ mm. $P = 16{,}077$

$\epsilon = +0.005$

$d_1 = 4.170$ $d_1^2 = 17.39$ $d_1^2 = 17.39$, $d_1^2 = 17.50$ } 17.445

$d_2 = 5.688$ $d_2^2 = 32.35$

$$\frac{d_1^2}{8R^2} = 0.00007304$$

For $\lambda=5085.824$ (*Cd*): $P=17{,}408$ $\quad \frac{d_1^2}{8R^2}=0.00005698$, $\epsilon=+0.000$

$17{,}408 \div 16{,}077.005 = 1.08278874$

1.00005698
$.000\ 7304$
0.99998394

$\lambda=5085.824\times 1.08278874\times 0.99998394$
$\lambda=5506.784$

ACCURACY OF RESULTS

The accuracy of wave-length determinations is largely dependent upon the degree of accuracy to which the diameters of the fringes may be determined. As a result of my experience I have found that successive readings on a well-developed inner ring may be repeated with an error not greater than 1 part in 500 to 600. However, this is not a fair test. My estimate of the accuracy of measurement is based upon a comparison of results obtained at an interval of about 24 hours (so as to eliminate the effect of visual memory) and also upon a calculation of the differences in the squares of the diameter of successive rings (which ought to be constant and equal to $\frac{8R^2}{P}$). As a result of these tests I am forced to the conclusion that the highest accuracy which may be claimed for the diameters of fringes used in the work here presented, is a little better than 1 part in 150, corresponding in the average to 0.004 Å. U.

From the table of numerical results it will be observed that the agreement between the results of different observers is fairly satisfactory. The results of Eversheim are also given, but a comparison between his results and the others is, perhaps, not just, as the greater part of his work was carried out on the more refrangible side of the green cadmium line where he himself had made no determinations of phase-change. Attention is to be called to the fact that while columns I, II, and IV, are based on Michelson's value of the wave-length of the red cadmium line ($\lambda=6438.4722$), column III is based upon the later determination of Fabry and Benoit ($\lambda=6438.4696$). In order to reduce the results of columns I and III to the same basis it is necessary to subtract about 0.0020 Å. U. from column I. Upon

making this change it is found that the difference between these two columns nowhere exceeds 0.004 Å. U.

In attempting to calculate the phase-change (ϵ) for the blue cadmium line, Michelson's values were substituted in expression (4) and it was invariably found that ϵ was greater than zero instead of less than zero. The fact that Lord Rayleigh and Eversheim have found similar irregularities would seem to point to the conclusion that Michelson's value for the blue cadmium line (4799.9107) is too small. It is to be considered that this line is not single but is made up of a complex of at least three lines and although Michelson's determination of the center of gravity of this complex at the separation of plates used by him, is undoubtedly correct, it seems inadvisable at the present time to use this line in accurate wave-length determinations.

The accuracy of the results here presented is by no means the highest of which the method is susceptible. The main fault to be found with the apparatus used, lies in the small dispersion of the grating and the small diameter of the inner fringes. Without going into these matters in detail, suffice it to say that the next step to be taken will consist in reconstructing the apparatus, using mirrors of longer focus, a grating of larger dispersion, and etalons giving the interferometer plates a greater separation. In addition to this, an attempt will be made to deposit films of speculum metal or magnalium by cathode discharge so that the fringes in the ultra-violet may compare in narrowness with those in the visible spectrum. By making these changes it is hoped that the wave-lengths of the iron lines may eventually be established to within 0.001 or 0.002 Å. U.

This investigation was made possible by a very generous grant from the Bache Fund, placed in the hands of Professor Ames, director of this laboratory, for the purpose of being devoted to this research.

JOHNS HOPKINS UNIVERSITY
May 1908

NOTES ON THE DETERMINATION OF THE ORBITS OF SPECTROSCOPIC BINARIES

By H. C. PLUMMER

1. The determination of the elements of a spectroscopic binary system is usually facilitated by the fact that observations have been made over a time long enough to fix the period with considerable accuracy and with sufficient care in the distribution of them to make known fairly precisely the extreme range in the variation of the radial velocity. With these data it is a fairly simple matter to determine from the velocity curve the other circumstances of the orbital motion, and a large number of practical methods have been proposed. It is needless to discuss their relative advantages: the method which gives the best results in one case may well be unsuitable in another. Under unfavorable circumstances any method by itself is liable to give untrustworthy results. As several methods are available which can be applied very easily and quickly, the best plan would seem to be to determine the approximate elements in several independent ways. The degree of accordance between the results so obtained will be a valuable guide to the judgment and may well save much labor which is sometimes spent in calculating trial ephemerides. It may therefore be useful to describe these methods briefly in a more connected and, incidentally, perhaps more general form than that in which they originally appeared. The notation adopted is as far as possible that which has come into general use.

2. The periods of spectroscopic binaries are most commonly short enough to be reckoned in days and the measured radial velocities are now invariably recorded in kilometers per second. Hence a natural and consistent system of units to adopt is:

Unit of time =1 mean solar day;
Unit of length =86400 km=0.000578 astronomical units;
Unit of velocity =1 km per second;
Unit of mass =that of the sun.

Velocities of approach are considered negative.

3. The radial velocity measured at any time t is

$$V=\gamma+Ke\cos\omega+K\cos u$$
$$=\gamma'+K\cos u,$$

where γ is the velocity relative to the sun of the center of mass of the system, K is the semi-amplitude of the velocity curve, u is the longitude in the orbit measured from the receding node in the direction of the motion, ω is the longitude of periastron, and e is the eccentricity. Let U be the period, and $\mu=2\pi U^{-1}$ the mean daily motion, and T the time of periastron passage. The quantities U, T, K, γ or γ', ω, and e may be regarded as the six elements which can be determined by spectroscopic observation. The inclination of the orbit cannot be so determined, while any possible information as to the size of the orbit and the masses can be deduced from the quantities defined.

4. The velocity curve may be referred to two axes, perpendicular to the axis of velocities, each of which has its advantages. The first is $V=\gamma'$ which lies midway between the maximum and minimum velocities. The maximum and minimum points A, B on the velocity-curve require $u=0°$ or $180°$; that is, they correspond to the receding and approaching nodes, respectively. The points where $V=\gamma'$ crosses the curve, E, F, require $u=90°$ and $270°$, that is, they correspond to the extremities of the focal chord at right angles to the line of nodes. The points A, E, B, F thus divide the velocity-curve into four parts corresponding to focal quadrants bounded by the line of nodes.

The second axis is the line $V=\gamma$ which, as was first pointed out by Rambaut, bisects the area of the velocity-curve. The points C, D where $V=\gamma$ crosses the curve require that the velocity relative to the center of mass shall be zero; that is, they correspond to the points on the orbit where the tangents are parallel to the line of nodes. The points $ACBD$ divide the curve into four parts such that the area from A to C equals that from C to B, and that from B to D equals that from D to the following maximum. These areal conditions impose a check on the drawing of the velocity-curve, especially in regard to the position of the maximum and minimum, and this reason alone makes it useful to determine the position of the line $V=\gamma$ by some planimetric process.

5. The points on the velocity-curve corresponding to certain im-

portant points of the orbit can be found very easily. Those corresponding to periastron and apastron, P_1, P_2, are determined by the following conditions: (1) their abscissae differ by $\frac{1}{2}U$; (2) their ordinates measured from $V=\gamma'$ are equal and opposite, for $u=\omega$ and $180°+\omega$; (3) the periastron point must lie in the shortest and the apastron point in the longest time interval in which the curve is divided at $AEBF$. The points L_1, L_2, corresponding to the extremities of the latus-rectum, are determined by these conditions: (1) they are equidistant in time from P_1, (2) their ordinates measured from $V=\gamma'$ are equal and opposite, for $u=\omega+90°$ and $\omega+270°$. And, generally, any pair of points which lie in alternate sections of the curve as determined by $AEBF$ and have equal and opposite γ'-ordinates correspond to the extremities of a focal chord. The points corresponding to the extremities of the minor axis, K_1, K_2, are determined by the conditions: (1) they are equidistant in time from P_1; (2) their ordinates measured from $V=\gamma$ are equal and opposite, for the corresponding velocities relative to the center of mass are parallel, equal, and opposite. Simple methods of finding the special points mentioned by means of the intersection of the curve with a suitably superposed copy have been given by Schwarzschild and by Zurhellen: they can, however, be found very easily by simple trial and error. The points C, D themselves correspond to the extremities of the diameter conjugate to the diameter which is parallel to the line of nodes since the corresponding orbital velocities are parallel to this line.

6. Let t_1 be the time at a point on the branch AEB of the curve and v and E_1, the corresponding true and eccentric anomalies. Let $180°+v$ and E_2 be the true and eccentric anomalies of the other end of the same focal chord, and t_2 the time at the corresponding point on the second part of the velocity curve. Then:

$$(1-e)^{\frac{1}{2}} \tan \tfrac{1}{2}v=(1+e)^{\frac{1}{2}} \tan \tfrac{1}{2}E_1 \; ; \quad \mu(t_1-T)=E_1-e \sin E_1 \; ;$$
$$-(1-e)^{\frac{1}{2}} \cot \tfrac{1}{2}v=(1+e)^{\frac{1}{2}} \tan \tfrac{1}{2}E_2 \; ; \quad \mu(t_2-T)=E_2-e \sin E_2 \; .$$

Hence

$$-(1-e)=(1+e) \tan \tfrac{1}{2}E_1 \tan \tfrac{1}{2}E_2 \, ,$$

or

$$e \cos \tfrac{1}{2}(E_2+E_1)=\cos \tfrac{1}{2}(E_2-E_1).$$

Consequently

$$\mu(t_2-t_1)=E_2-E_1-2e\sin\tfrac{1}{2}(E_2-E_1)\cos\tfrac{1}{2}(E_2+E_1),$$
$$=(E_2-E_1)-\sin(E_2-E_1);$$

from which E_2-E_1 can be found easily, especially with the help of the table given by Schwarzschild. Again

$$\tan\tfrac{1}{2}(E_2-E_1)=-\frac{(1-e^2)^{\frac{1}{2}}}{2e}(\cot\tfrac{1}{2}v+\tan\tfrac{1}{2}v),$$
$$=-\cot\phi\operatorname{cosec}v,$$

where $\sin\phi=e$.

7. Let now t_3, t_4 be the times at the ends of the perpendicular focal chord, so that the true anomalies are $v+90°$ and $v+270°$. The corresponding points on the velocity-curve can always be found by the conditions that they fall in the intermediate quadrants to the first pair and the sum of the squares of the γ'-ordinates of two points belonging to the different pairs is K^2. Then the two pairs will give:

$$\mu(t_2-t_1)=2\eta-\sin 2\eta;\quad \tan\phi\sin v=-\cot\eta$$
$$\mu(t_4-t_3)=2\eta'-\sin 2\eta';\quad \tan\phi\cos v=-\cot\eta'.$$

Hence we can find ϕ or the eccentricity and $v=u-\omega$. But this gives ω, for the γ'-ordinate corresponding to t_1 is $K\cos u$ and therefore gives u. Further, two independent values of T will be given by the equations

$$\tan\tfrac{1}{2}E_1=\tan(45°-\tfrac{1}{2}\phi)\tan\tfrac{1}{2}v;\qquad \mu(t_1-T)=E_1-e\sin E_1$$
$$\tan\tfrac{1}{2}E_3=\tan(45°-\tfrac{1}{2}\phi)\tan(45°+\tfrac{1}{2}v);\quad \mu(t_3-T)=E_3-e\sin E_3.$$

8. This general method can be applied to special pairs of points. Thus AB, EF are two such pairs, and in this case we have simply $v=-\omega$. The times of maximum and minimum velocity, however, are not always very exactly determined. Other two pairs of points are those for which the γ'-ordinates are $\pm K/\sqrt{2}$. In this case $u=45°$ and $v=45°-\omega$. Here the curve is usually steep and the corresponding times are well determined. Good results can accordingly be expected from this application of the method. Finally we have the points P_1P_2, L_1L_2, for which $v=0°$. Here the γ'-ordinates are $\pm K\cos\omega$, and $\pm K\sin\omega$, giving ω, and $t_1=T$. Also $\phi=\eta'-90°$, giving e. This is the elegant method of Schwarzschild as simplified by Nijland and Zurhellen. In this case there is no independent determination of T, the last equation of the preceding paragraph reducing in effect to $2T=t_3+t_4$.

9. From the properties of a focal chord we may turn to the properties of a diameter of the orbit. Let t_1, t_2 be the times of passing the ends of a diameter and let $E+\frac{1}{2}\pi$, $E+\frac{3}{2}\pi$ be the corresponding eccentric anomalies, E being the eccentric anomaly of the end of the conjugate diameter. Then

$$\mu(t_1-T)=E+\tfrac{1}{2}\pi-e\cos E,$$
$$\mu(t_2-T)=E+\tfrac{3}{2}\pi+e\cos E,$$

whence

$$\tfrac{1}{2}\mu(t_1+t_2-2T)=E+\pi,$$
$$\mu(t_2-t_1-\tfrac{1}{2}U)=2e\cos E.$$

Now it has been seen that C, D, where the curve meets the γ-axis, are such a pair of points, the conjugate diameter being parallel to the line of nodes. Hence

$$-\tan\omega=\cos\phi\tan E,$$
$$=\cos\phi\tan\tfrac{1}{2}\mu(t_1+t_2-2T),$$

and

$$-e=\tfrac{1}{2}\mu(t_2-t_1-\tfrac{1}{2}U)\sec\tfrac{1}{2}\mu(t_1+t_2-2T),$$

as given by Zurhellen; and by eliminating T,

$$\tfrac{1}{2}\mu(t_2-t_1-\tfrac{1}{2}U)=e\cos\phi\,(\cos^2\phi+\tan^2\omega)^{-\frac{1}{2}},$$
$$=e\cos\omega\,(1-e^2\cos^2\omega)^{-\frac{1}{2}}\cos\phi,$$

as given by Lehmann-Filhés for the determination of ϕ or e when $e\cos\omega$ has already been determined and e is not small. Again K_1, K_2, the ends of the minor axis, are such a pair of points. Here $E=0$, and hence

$$e=\tfrac{1}{2}\mu(t_2-t_1-\tfrac{1}{2}U).$$

Also the radial components of the velocities at the ends of the minor axis are $\pm K\cos\phi\sin\omega$, and these quantities will therefore be given by the γ-ordinates at K_1, K_2. These properties have also been given by Zurhellen.

10. Let z_1, z_2 be the areas, taken positively, between the curve and the γ-axis from A to C and from B to D respectively. These are the projections on the plane containing the line of sight and the line of nodes of the perpendiculars from C and D in the orbit on the line of nodes. Hence we obtain

$$\tfrac{1}{2}(z_1+z_2)=K\mu^{-1}\cos\phi\,(1-e^2\cos^2\omega)^{\frac{1}{2}},$$
$$\tfrac{1}{2}(z_1\sim z_2)=K\mu^{-1}\cos\phi\cdot e\sin\omega.$$

The method of Lehmann-Filhés uses only the ratio of these quantities and the unit in which the areas are measured is immaterial. But by

measuring a rectangle with given sides it is easy to find the planimeter value for an area of 1 km/sec. by 1 day and so to evaluate the areas in the proper units. Then we have e determined by the equation

$$z_1 z_2 = K^2 \mu^{-2} \cos^4 \phi ,$$

and ω is determined by the second of the previous equations. Thus ω and e can be determined directly from the areal properties of the velocity-curve, and we also have

$$\gamma' - \gamma = Ke \cos \omega .$$

11. If ψ is the angle between the tangent to the velocity-curve and the time-axis,

$$\tan \psi = \frac{dV}{dt} = -K \sin u \frac{dv}{dt}$$
$$= -\mu K \cos \phi \sin u \cdot a^2 r^{-2} .$$

where a is the mean distance in the actual orbit and r is the radius-vector of the corresponding point. Now cos u is given by the γ'-ordinate of the point on the velocity-curve. Hence by drawing tangents to this curve at a number of points it would be quite possible to reconstruct the actual orbit on an arbitrary scale by the polar co-ordinates of its points: for

$$v = u - \omega ; \quad r \propto (\sin u \cot \psi)^{\frac{1}{3}} .$$

The method seems practical but rather long: its interest lies in the analogy to Sir John Herschel's method for double stars by which, through distrust of the measured distances, he constructed the apparent orbit from the position-angles alone. Without going so far as to construct the orbit in this way, however, useful information may be obtained from the direction of the velocity-curve at special points. Zurhellen has shown how the eccentricity can be derived from the directions at the periastron and apastron points, which is only one particular case. We have in general

$$\tan \psi = -\mu K \sec^3 \phi \sin u (1 + e \cos v)^2 ;$$

and thus at

A, B:	$u = 0°, 180°$;	$\tan \psi = 0$
E, F:	$u = 90°, 270°$;	$\tan \psi = \mp \mu K \sec^3 \phi (1 \pm e \sin \omega)^2$.
P_1, P_2:	$u - \omega = 0°, 180°$;	$\tan \psi = \mp \mu K \sec^3 \phi \sin \omega (1 \pm e)^2$.
L_1, L_2:	$u - \omega = 90°, 270°$;	$\tan \psi = \mp \mu K \sec^3 \phi \cos \omega$.
K_1, K_2:	$u - \omega = 90° + \phi, 270° - \phi$;	$\tan \psi = \mp \mu K \cos \phi \cos (\omega \pm \phi)$.

The angle ψ is correctly obtained from the curve only when it is drawn to the same scale in time and velocity, e. g., in the units 1 day and 1 km/sec. If a different scale is used in the two directions, allowance must be made for this fact in finding tan ψ, although it is not material when only the ratio of two such expressions is used.

12. When the eccentricity is small, the velocity-curve approaches in form the simple sine-curve and graphical methods cease to be of value. It is then that the analytical method becomes useful. The observations may be represented by a series

$$V=b_0+b_1\cos\mu t+b_2\cos 2\mu t \\ +a_1\sin\mu t+a_2\sin 2\mu t\,,$$

the coefficients in which can be calculated either by some method of harmonic analysis or by least squares. The period being supposed known, the five terms are all that are required to determine the other five elements of the orbit. Now

$$\begin{aligned} V&=\gamma+K(e\cos\omega+\cos u)\,,\\ &=\gamma+K\cos^2\phi\cos\omega\cdot\frac{\cos E}{1-e\cos E}-K\cos\phi\sin\omega\cdot\frac{\sin E}{1-e\cos E}\,,\\ &=\gamma+K\cos^2\phi\cos\omega\sum_{j=1}c_j\cos jM-K\cos\phi\sin\omega\sum_{j=1}s_j\sin jM\,, \end{aligned}$$

where $M=\mu(t-T)$ is the mean anomaly. Here

$$\begin{aligned} c_j&=\frac{1}{\pi}\int_0^{2\pi}\frac{\cos E}{1-e\cos E}\cos jM\,dM\,,\\ &=\frac{1}{\pi}\int_0^{2\pi}\cos E\cos(jE-je\sin E)dE\,,\\ &=J_{j+1}(je)+J_{j-1}(je)=2e^{-1}J_j(je)\,;\\ s_j&=\frac{1}{\pi}\int_0^{2\pi}\frac{\sin E}{1-e\cos E}\sin jM dM\,,\\ &=\frac{1}{\pi}\int^{2\pi}\sin E\sin(jE-je\sin E)dE\,,\\ &=J_{j-1}(je)-J_{j+1}(je)=\frac{2d\cdot J_j(je)}{d(je)}\,, \end{aligned}$$

where J_j is a Bessel's function of order j.

$$J_j(x)=\frac{x^j}{2^j j!}\left\{1-\frac{x^2}{2(2j+2)}+\frac{x^4}{2\cdot4\cdot(2j+2)(2j+4)}-\cdots\right\}$$

Hence by comparison with the series of which the coefficients have been calculated, $b_0=\gamma$ and

$$a_j=2e^{-1}K\cos^2\phi\cos\omega\,J_j(je)\sin j\mu T-2K\cos\phi\sin\omega\,J'_j(je)\cos j\mu T;$$
$$b_j=2e^{-1}K\cos^2\phi\cos\omega\,J_j(je)\cos j\mu T+2K\cos\phi\sin\omega\,J'_j(je)\sin j\mu T.$$

The four equations obtained by making $j=1$ and $j=2$ will give K, T, e, and ω by successive approximations. When the first power of e alone is retained, $J_1(e)=\frac{1}{2}e$, $J_2(2e)=\frac{1}{2}e^2$, $J'_1(e)=\frac{1}{2}$, $J'_2(2e)=\frac{1}{2}e$ and $\cos\phi=1$. Hence the equations in this case reduce to those given by Wilsing:

$$a_1=-K\sin(\omega-\mu T),\quad b_1=K\cos(\omega-\mu T);$$
$$a_2=-eK\sin(\omega-2\mu T),\quad b_2=eK\cos(\omega-2\mu T).$$

The first approximation to e is therefore given by

$$e^2=(a_2^2+b_2^2)\div(a_1^2+b_1^2).$$

The development of further approximations has been discussed by Russell. Except when the eccentricity is small this method offers in general no advantage over the graphical methods.

13. When approximate values of the elements have been obtained by any of the foregoing methods, or from any combination of them, the good quality of the observations may justify a final solution by the method of least squares. The form of the equations of condition has been given by Lehmann-Filhés. For completeness they are given here in what seems a slightly simplified form. Let n be the number of periods over which the observations extend, and let

$$w=-K\delta\omega,\quad \epsilon=-K\sec^2\phi\,\delta e,$$
$$\tau=\mu K\sec^3\phi\cdot\delta T,\quad \nu=-nKU\sec^3\phi\cdot\delta\mu.$$

Then the equations of condition become

$$V_0-V_c=\delta\gamma'+\cos u\,\delta K+\sin u\cdot w$$
$$+\sin u\sin v\,(2+e\cos v)\cdot\epsilon$$
$$+\sin u\,(1+e\cos v)^2\tau+\sin u\,(1+e\cos v)^2\frac{t-T}{nU}\nu.$$

In this form no further transformation seems necessary in order to secure homogeneity in the coefficients. After the corrections to the elements have been found, γ can be recalculated from the new values since its correction is not included explicitly in the equations of condition.

14. When the elements as already defined have been determined, some information in regard to the size and masses of the system can be deduced, but only in terms of the inclination, which spectroscopic observations alone leave unknown. Let $i(<90°)$ be the angle between the line of sight and the normal to the plane of the orbit. Then

$$a \sin i = K\mu^{-1} \cos \phi \cdot 86400 \text{ km} ,$$
$$m'^3(m+m')^{-2} \sin^3 i = K^3\mu^{-1} \cos^3 \phi \cdot [3.81445-10],$$

where the sun's mass is unity, m is the mass of the star whose radial velocity has been observed, and m' is the mass of the other star. If the radial velocity V' of the second component of the binary system is measured at the same time,

$$m(V-\gamma)+m'(V'-\gamma)=0 .$$

One such equation will give $m:m'$ when γ is known and two will give γ in addition without any knowledge of the orbit. It is assumed that the velocities have been determined by means of a comparison spectrum; if no comparison spectrum has been used and only the relative velocities of the two components have been determined from the positions of corresponding lines in the two superposed spectra, by a' must be understood the mean distance in the relative orbit instead of the mean distance of one component about the center of mass, and for the expression $m'^3 (m+m')^{-2}$ above must be substituted $(m+m')$.

15. The relation of spectroscopic observations to the visual orbit may be briefly considered. If visual observations have been made over a sufficiently long time to determine the elements, the inclination i is known and also the mean distance a'' expressed in seconds. Let a and a' be the mean distances of the components about the center of mass, expressed in the linear unit adopted, 86400 km, so that

$$ma = m'a' = \frac{mm'}{m+m'}(a+a')$$
$$= \frac{mm'}{m+m'} \cdot \frac{a''}{\pi''} \cdot [3.23813] ,$$

where π'' is the parallax. Hence

$$V = \gamma + K(e \cos \omega + \cos u)$$
$$= \gamma + \mu a \sec \phi \sin i(e \cos \omega + \cos u)$$
$$= \gamma + [3.23813] \cdot \mu \sec \phi \sin i(e \cos \omega + \cos u) \cdot \frac{a''}{\pi''} \cdot \frac{m'}{m+m'} ;$$

and similarly for the second component

$$V'=\gamma-[3.23813]\cdot\mu\sec\phi\sin i(e\cos\omega+\cos u)\cdot\frac{a''}{\pi''}\cdot\frac{m}{m+m'}.$$

Thus if the spectrum of only the first component can be measured at different times, it is only possible to determine γ and $(1+m/m')\,\pi''$. The relative velocity alone

$$V-V'=[3.23813]\cdot\mu\sec\phi\sin i(e\cos\omega+\cos u)\,a''/\pi''$$

will give the parallax and the sum of the masses, for

$$m+m'=[3.52884]\mu^2(a''/\pi'')^3.$$

But if the velocities of the two components are determined by reference to a comparison spectrum the ratio of the masses and hence the masses themselves, compared with the sun, can also be determined.

16. The ambiguity regarding the position of the plane of the orbit as determined by visual observations can be removed, as is well known, only by measures of the radial velocity. It will be supposed that i, as previously defined, is a positive angle less than 90°. Let m be the mass of the principal star and m' that of the companion. There is much confusing variety in the notation employed by the computers of double-star orbits, but we may assume the system in which ρ, θ, the distance and position-angle, are given by

$$\rho\cos(\theta-\Omega)=r\cos(v+\lambda),$$
$$\rho\sin(\theta-\Omega)=\pm r\sin(v+\lambda)\cos i,$$

where Ω is the position-angle of the node falling between 0° and 180°, λ is the longitude of periastron measured from this node in the direction of the orbital motion; and as regards the double sign the upper corresponds to direct and the lower to retrograde motion of the companion. The question now is whether λ and ω are identical or whether they differ by 180°? The data required for the answer may be either (1) V_1, V_2, the radial velocities of the principal star at different times, or (2) V, V', the radial velocities of the two stars at the same time. Then it is easily seen that if

$$V_1-V_2 \text{ and } \cos(v_1+\lambda)-\cos(v_2+\lambda)$$

or if

$$V-V' \text{ and } e\cos\lambda+\cos(v+\lambda)$$

are of the same sign, $\lambda=\omega$; if of opposite sign, $\lambda=\omega+180°$. In the former case the receding node of the principal star corresponds

to the companion at Ω, i. e., the companion is approaching the observer when at its first node; in the latter case the companion is receding at Ω. This way of regarding the question differs slightly from the point of view adopted by Dr. Campbell (*Lick Observatory Bulletin*, No. 70) but it seems to involve less disturbance of the system of elements universally adopted by the spectroscopists on the one side and that adopted by the majority of the computers of visual orbits on the other.

17. Reference has been made in the foregoing to the following papers:

Rambaut, *Monthly Notices*, **51**, 316, 1891.
Lehmann-Filhés, *Astronomische Nachrichten*, **136**, 17, 1894.
Schwarzschild, *ibid.*, **152**, 65, 1900.
Nijland, *ibid.*, **161**, 103, 1903.
Zurhellen, *ibid.*, **175**, 245, 1907.
Wilsing, *ibid.*, **134**, 89, 1894.
Russell, *Astrophysical Journal*, **15**, 252, 1902.

In addition to these may be mentioned the recent paper by King (*Astrophysical Journal*, **27**, 125, 1908), and for a more detailed account of the better known methods, that by H. D. Curtis (*Pub. A.S.P.*, **20**, 133, 1908).

Mt. Hamilton
May 1908

SERIES IN THE SPECTRUM OF BARIUM

By F. A. SAUNDERS

The spectrum of barium has been studied by Kayser and Runge (*Wied. Ann.*, **43**, 385, 1891), Rydberg, Exner and Haschek, Hermann (*Annalen der Physik*, **16**, 684, 1905), and others. Kayser remarks (*Handbuch der Spectroscopie*, **2**, 541) that no series have been found in this spectrum, though several triplets were noticed. Rydberg (*Wied. Ann.*, **52**, 126, 1894) mentions such triplets, and gives the proper values for their "width" on a wave-number scale. I have not, however, been able to find any published work showing the wave-lengths of the lines of these triplets, nor how they are arranged in series. Since the triplets in the spectra of calcium and of strontium are conspicuous, one would naturally expect to find them easily in barium. That they have not been found long ago is in itself proof that they are neither very strong nor very striking.

The writer has taken a good set of photographs of the spectrum of barium, extending from λ 8000 to λ 1900. A parabolic concave grating[1] was used to λ 2300 and a quartz spectrograph for the rest. The arc was the main source of light, though some spark spectra were also obtained for comparison. Graphite rods were used as terminals, and the best barium chloride obtainable on the market as the source of barium. The graphite rods contain *Ti* in small quantities, and spectroscopic traces of *Al*, *Bo*, *Cu*, *Si*, *Ca*, and *Na* were nearly always present also. The barium salt contributed *Sr* to the list of impurities. The presence of these foreign substances was rather an advantage than otherwise, as their lines served as standards, being always sharp, and they were by no means so numerous as to be in the way. None of the new lines added to the barium list is sharp; they cannot therefore be confused with the lines due to impurities, and, indeed, the excellent lists of Exner and Haschek make such confusion very unlikely. Check photographs of the spectra of the impurities were

[1] This equipment is described in the *Astrophysical Journal*, **20**, 188, 1904. It is a pleasure to express again my indebtedness to the Rumford Fund Committee for the grant which made it possible to purchase this magnificent instrument.

used for comparison, when necessary. The great cyanogen bands were very much in the way, and I was forced to take several photographs, using the barium salt on rods of *Cu* or *Al* in order to be able to find the fainter lines in these regions.

My red-end photographs show many lines that were new at the time they were measured. The excellent measurements of Hermann in this region make it unnecessary for me to publish my figures at present. Only three red lines are used in the series arrangement; my measurements for these gave λ 7906.0; 7392.6; and 7195.5; Hermann gives λ 7906.13; 7392.83; and 7195.71. The agreement is, under the circumstances, satisfactory. A complete list of the lines of the barium spectrum was prepared, the wave-lengths reduced to vacuum, and the reciprocals taken. These wave-numbers were then plotted on a large scale. To find triplets of constant frequency-difference reduced itself to finding lines at constant distances apart, and the work was made much easier.

With the information thus available, it proved to be easy to find three complete triplets of the second subordinate series, and fragments of two more followed later. Analogy then led me to search for a strong member of the first subordinate series in the green, but here a difficulty was encountered. The only possible group included the great line λ 5535 as one of its members. But this was clearly out of the question, since this is a Bunsen flame line, totally different in its characteristics from the typical series lines. A careful examination of this line under fairly high dispersion made it seem likely that it was double; that is, that the line I sought was present beside the line λ 5535. The new line was too faint to be seen when λ 5535 was sharp, since this occurs only with very small amounts of barium in the arc; and, when a large quantity of barium was introduced, the stronger line spread out so far that it obliterated the weaker one entirely. The best conditions for observing the line are therefore when λ 5535 is of medium strength, and feebly reversed, if at all. It then appears and behaves like its neighbor λ 5519, which belongs to the same series triplet. It was not possible to regulate the quantity of barium so exactly as to be able to keep these conditions constant and photograph the new line. Feeling that the dispersion available was not sufficient for exact eye measurements, I asked Professor N. A.

Kent of Boston University, who is working with a 21-foot grating, if he would kindly examine λ 5535 for me. Without being informed of the position of the new line, he independently discovered it, and measured its wave-length in the second-order spectrum, with the result 5536.07 (the main line being 5535.69). This agrees almost exactly with the value which I had in the meantime predicted for it from the triplet spacing. Professor Kent took great pains to identify the lines near by and thus to show that the new line really belonged to barium. For the very generous manner in which he placed his time at my disposal, and for the great care and skill with which he made these observations, I wish to record my sincerest gratitude.

This first subordinate triplet being then complete, there was little difficulty in tracing out the others. None of them is so prominent as one might expect, and some of the necessary lines had escaped previous observation through faintness, or by hiding behind the cyanogen bands.

The two subordinate series are tabulated below, arranged according to Rydberg's scheme, so as to show the constancy of wave-number differences. All the wave-numbers are corrected to vacuum.

TABLE I

FIRST SUBORDINATE SERIES OF BARIUM

λ	$1/\lambda$	ν	λ	$1/\lambda$	ν	λ	$1/\lambda$
5819.21	17179.8	878.0	5536.07	18058.4	370.4	5424.82	18428.8
	55.4			54.7			
5800.48	17235.2	877.9	5519.37	18113.1			
	127.6						
5777.84	17302.8						
			4333.04	23072.2	371.0	4264.45	23443.2
				52.7			
4493.82	22246.7	878.2	4323.15	23124.9			
	21.4						
4489.50	22268.1						
			3947.6	25325.0		A	
				12.8			
4087.53	24458.1	879.7	3945.6	25337.8			
	15.3						
4084.94	24473.4						
3895.2	25665.4	870	3767.5	26536		B	
3787	26399						

TABLE II

SECOND SUBORDINATE SERIES OF BARIUM

λ	1/λ	ν	λ	1/λ	ν	λ	1/λ
7906.0	12645.0	878.4	7392.6	13523.4	370.3	7195.5	13893.7
4903.11	20389.6	878.3	4700.64	21267.9	370.3	4620.19	21638.2
4239.91	23578.9	879.2	4087.53	24458.1	370.1	4026.57	24828.2
3975.55	25146.8		C				
3841.72	26022.8						

The columns of these tables which are headed ν contain the wave-number differences, which should be constant. The numerous gaps are due to the excessive faintness of the lines. The lines toward the end of the first series are nearly all new, but their wave-lengths are very inaccurate. The line λ 3841 of the second series is also new. The line λ 4087 appears to belong to both series; calculation shows that two lines occur here, only 0.4 Å apart; since both are diffuse, I could not separate them. At the place marked *A*, the line λ 3890.64 is calculated to appear, but the strong barium lines 3891.97 and 3889.45 blot it out. At the place *B* there is a faint haze on some of my photographs, but it is not measurable. At the place *C* a line is due whose wave-length is within 0.4 Å of the line 3841; it cannot be seen separately.

FORMULAE FOR FIRST SUBORDINATE SERIES

In calculating the equation of this series, there is a choice of lines to be used. I have taken the following wave-numbers: 17180.0; 22194.4, 24447, 25659, and 26395. These I have obtained by calculation from observed lines in the triplets as the most probable values for the line of greatest wave-length in each triplet. Unfortunately this line is missing in all but the first triplet; but, since it heads the tabulation on Rydberg's scheme, it seems to be analogous in position to the first line of the second subordinate triplets, and therefore to be the best to use in the calculations.

Kayser and Runge's formula cannot be used to represent the first subordinate series of *Ba* with very satisfactory results. Rydberg's type suits it better, and the formula runs as follows:

$$1/\lambda = 28300.2 - \frac{91015.6}{(m+.8609)^2}.$$

The errors (calc. λ − obs.) for the various lines are as follows:

0.0 0.0 −0.17 +1.2 +4.0

A better agreement is given by the formula,

$$1/\lambda = 28481 - \frac{109675}{(m+.37045)^2 - 1.655},$$

which contains three adjustable constants, the number 109675 being the universal series-constant used by Ritz. The errors with this formula are:

0.0 0.0 +0.02 −0.7 −0.5

This is entirely satisfactory. The calculated first term would be near 12 μ.

FORMULAE FOR SECOND SUBORDINATE SERIES

Kayser and Runge's type fails conspicuously with this series when the first (and most important) line is included; the same thing happens also in the case of *Ca* and *Sr*. Rydberg's is very little better. It runs as follows:

$$1/\lambda = 28466.6 - \frac{99091}{(m+.5026)^2}.$$

The errors (as above) are:

0.0 0.0 0.0 −7.5 −15. Å

The observations for the fourth line are accurate to 1 Å. These errors are therefore intolerable.

Ritz[1] has proposed two types of formulae (with three adjustable constants), neither of which fits this series as well as Rydberg's. I have made a lengthy search for any modification of any of these formulae which would suit this series better without introducing a fourth constant, but so far without success. This result is the more unexpected since Ritz's formula is quite successful with the second subordinate series of *Ca* and *Sr*, but a careful review of the observations and calculations shows no other alternative. I am therefore reluctantly driven to make use of a four-constant formula. Of the many possible types, I have calculated out the following one:

$$1/\lambda = 28062 - \frac{73392.7}{(m+.9027)^2 + 1.1403}.$$

This gives practically perfect agreement for all five lines; four of them were, of course, used for the calculation of the constants.

[1] *Annalen der Physik*, 12, 264, 1903.

The spectra of *Ca*[1] and *Sr*[2] contain groups of narrow triplets which form at least one series, and probably two. These are present in *Ba* also; Rydberg gave their correct wave-number differences, but not their positions. The more important series contains the following lines, and is of the first subordinate type, i. e., complex:

TABLE III

NARROW TRIPLETS SERIES

1/λ	ν	1/λ	ν	1/λ
25018.6 25033.1	381.7	25386.0 25400.3	182.1	25568.1
29227.5	374.6	29602.1	178.0	29780.1
31017.2	376.5	31393.7		
32047.6(?)				

Most of these lines are diffuse and faint, and I do not think that the observations are complete enough at this time to make it worth while to fit a formula to this series. I hope to be able to report on this at a later date, as also upon the series of pairs in this spectrum, whose existence was first shown by observations on the Zeeman effect.

It is a pleasure to record my indebtedness to Mr. J. H. Morecroft, now of the Pratt Institute, Brooklyn, for the very great assistance which he gave me, while a student in this university, in the laborious task of plotting this spectrum, and searching for regularities in it.

SYRACUSE UNIVERSITY
June 1908

[1] *Astrophysical Journal*, 21, 195, 1905.
[2] *Ibid.*, 21, 81, 1905.

THE SPECTRUM OF COMET *d* 1907 (DANIEL)

By W. W. CAMPBELL

The spectrum of Comet *d* 1907 was observed on several mornings between July 14 and September 8, 1907, with the one-prism spectroscope described in *Lick Observatory Bulletin* No. 8 and illustrated in *Bulletin* No. 62, as attached to the 36-inch refractor.

On July 14 the spectrum was observed visually. The carbon bands whose edges are at λ 4737, λ 5165, and λ 5635 were present in their usual relative intensities, but the continuous spectrum of the nucleus and coma was relatively strong. The nucleus, conspicuous with low power, was so large and weak under the higher magnification of the 36-inch refractor, that it was not worth while exposing for the photographic spectrum.

By August 9 the character of the visual spectrum had changed considerably. The continuous spectrum of the nucleus remained strong. The spectrum of the coma consisted principally of the three carbon bands, that at λ 5165 showing two or three maxima corresponding to the heads of component flutings, but the continuous spectrum between the bands was faint. The bands seemed to cross the nucleus without perceptible strengthening at the nucleus. The bands were followed out into the tail for about 1°, but they grew fainter with increased distance from the nucleus, and fainter relatively to the continuous spectrum. Beyond 1° the spectrum of the tail was faint, and it was not certain that the bands were present, though the continuous spectrum could still be seen. Dr. Albrecht took part in the observations on this night, and his results were in harmony with mine.

As the comet approached perihelion, on September 4, the bright bands appeared to develop in relatively greater strength.

Exposures for spectrograms were made on nine mornings. One of two hours on July 28 and another of sixty-five minutes on August 28 recorded the strongest lines but faintly. An exposure of two hours with the three-prism Mills Spectrograph, made in the hope of record-

ing the brighter lines of the blue carbon band, as a basis for determining their wave-lengths very accurately, did not succeed; and an exposure of ninety minutes on August 15 with the one-prism instrument, using a wide slit directed upon the tail at a point $\frac{3}{4}^\circ$ from the nucleus, likewise gave no record. The other five exposures were very successful. The table below contains the wave-lengths of bright lines obtained from the five plates, as well as Kayser and Runge's wave-lengths of prominent lines in the carbon and cyanogen spectra,[1] and the wave-lengths of lines observed by me in Comets *c* 1893[2] and *b* 1894[3] and by Wright in Comet *a* 1899.[4] The first line of headings in the table gives the astronomical date of observation, the second line the exposure time, and the third the name of the observer who measured the plate. The spectrogram of August 22, reproduced in an enlarged form in the accompanying Plate XV, was purposely made without comparison spectrum in order that a very long slit might be used. To obtain wave-lengths for the lines on this plate it was necessary to assume the positions of the comet bright lines at λ 3871.5, λ 4737.2, and λ 5165.3, which are printed in italics in the table, and of the comet dark lines at λ 4101.89 ($H\delta$) and λ 4383.72 (*Fe*). The spectrogram of August 16 is on a Cramer's isochromatic plate, which accounts for the greater extension in the yellow. This spectrogram is also reproduced, with iron comparison spectrum.

Of the sixty-one bright lines catalogued in the table, a few are possibly unreliable, for the reason that they are involved with the comet's continuous and dark-line spectrum as a background, and one cannot always determine whether a certain appearance is due to a bright line or to a continuous spectrum between dark lines. In such cases the interpretations made by two observers will easily and unavoidably differ, as the table illustrates; but it appears wise to include all lines measured by each observer. They will be of interest and probable value in connection with similar observations in the future.

The observations have been corrected for the relative orbital

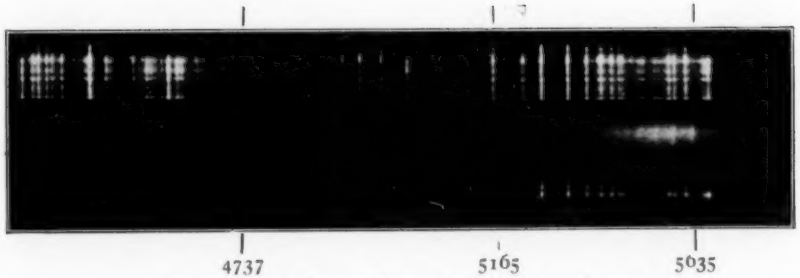
[1] *Anhang zu Abhandlungen der K. Akad. der Wiss.*, Berlin, 1889.

[2] *Astronomy and Astrophysics*, 12, 652, 1893.

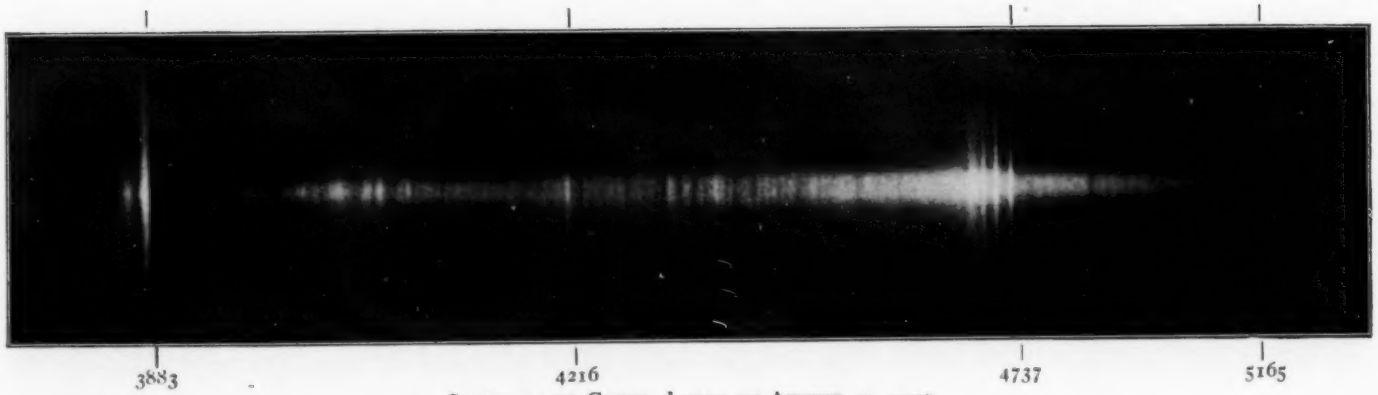
[3] *Astronomical Journal*, 14, 111, 1894.

[4] *Astrophysical Journal*, 10, 174, 1899.

PLATE XV



SPECTRUM OF COMET *d* 1907 ON AUGUST 16, 1907



SPECTRUM OF COMET *d* 1907 ON AUGUST 22, 1907

motions of the comet and earth. The motions relative to the sun and earth at the times of observation were:

		Comet–Sun	Comet–Earth
1907	August 10.....	−28 km	+14 km
	August 13.....	−27	+19
	August 16.....	−25	+23
	August 22.....	−20	+29
	September 8.....	+10	+36

In forming the mean values of wave-lengths each measure was treated as one observation: in observations of this kind, personal errors of interpretation as to the point to be measured are larger than plate errors.

There can be no doubt that the second, third, fourth, and fifth bands of the arc spectrum of carbon and the second and third bands of cyanogen are represented in the comet. In fact, they correspond very nearly to the whole of the comet's photographic bright-line radiations. There are no identifications of other elements or compounds. In particular, no trace of hydrogen bright lines could be seen, unless the very faint line at λ 3890 measured by one observer on one plate is $H\zeta$, which is improbable.

A Plücker tube spectrum of cyanogen, obtained for comparison with the same instrument, recorded a large number of strong bands throughout the spectrum, but those beginning at λ 3872 and λ 3884 are much the faintest bands within several hundred Ångström units of their positions. There appear to be no other points of resemblance or coincidence between the comet spectrum and this tube spectrum.

A spark spectrum of carbon was obtained for the same purpose. Subtracting the lines of iron, which element appears to be present as an impurity, nearly all the light falls in the third cyanogen band, beginning at λ 3884. The second cyanogen band is also recorded, but not nearly so strongly as the third. There are apparently no other points of resemblance to the comet spectrum. The measured wave-lengths obtained from the one-prism carbon spark, and Kayser and Runge's carbon arc results, are:

Carbon Spark	K. and R. Arc
3850.96	
54.66	3855.06
61.94	61.86
71.60	71.54
83.63	83.55
4152.36	4152.88
58.05	58.17
68.16	67.77
81.27	80.98
97.38	97.24
4216.36	4216.12

It is well known[1] that one cannot determine accurately the position of the edge of an unsymmetrical band, such as those here dealt with. The intensities of the individual lines composing the band, the width of the slit used, the exposure-time, etc., all affect the apparent position of the edge. Granted the monochromatic character of the lines making up the band's edge, the observer's best practice is to set the micrometer wire at a distance within the edge equal to one-half the width of the image of a neighboring comparison line whose intensity is estimated to equal the intensity of the edge of the band. A minor source of difficulty still remains, in that a symmetrical line is compared with one edge of a wide band, and slight error is scarcely avoidable.

The band at $\lambda\lambda$ 3884–3872 produced by the carbon spark and by the *CN* tube do not reproduce at all closely the relative intensities within the comet band. It is well to recall that a hydrogen tube used by Wright in studying the spectrum of Comet *a* 1899[2] gave not only the hydrogen lines but also the bands of cyanogen at λ 3884 and λ 3872 apparently exactly as they appeared in the spectra of that and all the other comets showing these bands on our spectrograms.

The reproductions of the spectrograms dated August 16 and August 22, 1907, are fairly satisfactory, though considerable detail is lost, especially in the region λ 3884–λ 3855, and the relative intensities are altered here and there. The index lines below and above are displaced in a few cases, apparently because the engraver, distant in the eastern part of the country, was not able to work more accurately to my lines on the proof sheet.

[1] See Kayser's article in *Astronomy and Astrophysics*, **13**, 367, 1894.

[2] *Astrophysical Journal*, **10**, 174, 1899.

The continuous and dark-line spectra are shown on all the plates measured, but especially well on that of August 22. In the latter case the slit extended far on either side of the comet's nucleus, and there is no possibility that the sky spectrum of dawn exerted any influence on the result. For the spectrograms of August 10 and September 8 the continuous and dark-line spectra are strongly recorded also, but the slit on both occasions was relatively short, to permit the impressing of iron comparison spectra, and there is a doubt as to whether the sky spectrum combined with the comet spectrum and to what extent.

The continuous and dark-line spectrum of the nucleus of the reproduced plate of August 22 appears to be a duplicate of the solar or Fraunhofer spectrum, if we make allowance for the superimposed bright-line spectrum, though I strongly suspect the maximum of continuous spectrum to be displaced toward the red. However, the carbon band at λ 4737, lying near the usual maximum, complicates the question. The long bright lines cross the spectrum of the nucleus with less intensification at the nucleus than the half-tone seems to show: as if they were due almost wholly to coma radiations, the nucleus contributing to them but feebly. Many of the short bright lines, however, seem not to extend beyond the nucleus, though in many cases this may be due largely to their faintness.

The nucleus of the comet is evidently shining almost entirely by virtue of sunlight reflected by the materials composing it. Several short bright lines observed are evidence of considerable weight that there are some bright-line radiations from the nucleus. It is not impossible that a great many short lines are lost to view in the strong continuous spectrum.

The coma is evidently shining very largely by inherent light, though the faint continuous spectrum observed visually but not showing on the spectrogram would perhaps contribute appreciably to the ordinary integrated photograph of the comet.

The spectroscopic observations of the tail seem to show that the inherent light, existing in large proportion near the head, decreases in proportion to reflected or diffused sunlight with increasing distance from the head.

Independent observations by Mr. Duncan and myself showed slight

BRIGHT LINES IN COMET, CARBON, AND

1907, Aug. 10 85 minutes			Aug. 13, 94 Minutes		Aug. 16 90 Min.	Aug. 22, 65 Min.	Sept. 8 30 Minutes			Descriptions
Campbell	Plummer	Duncan	Campbell	Duncan	Campbell	Campbell	Campbell	Plummer	Duncan	
						3854.8				very faint, poorly defined
3867.0						62.4				poorly defined
70.1						68.2				very bright, long (clearly
74.1	3873.5		3870.			3871.5				very bright, long) resolved
76.9						74.6				well defined
79.7						77.3				bright, not well defined
			79.5	80.1	3879.1	80.5	3879.3			brightest lines on plate, long, clearly double
82.2	81.8	82.6	81.9	82.4	82.0	82.5	82.8	3882.5	82.0	brightest lines on plate, long, clearly double
	89.5									not full length of slit
						3991	3988.4			bright, short, poorly defined
						4003	4003.5	4003.5		bright, short, poorly defined
4013.9										bright, short, broad
20.2						20.4	19.3			bright, short, broad
39.4			38.8			39	38.9			bright, short, broad
42.6	42.4		42.1			44	43.0			bright, short, broad
51.0			49.9	51.5		51	50.9			bright, close double
	52.7		53.0			53				bright, close double
						65.3	65.3			fairly bright
68.1						68.9	69.5			fairly bright
74.2	74.2					73.7	{ 73.0 } { 74.9 }			bright, close double?
								4165.8		doubtful
								69.2		broad
								79.6		distinct
								85.1		clear
								89.6		clear
							4193.2	93.1		fairly bright, well defined
4214.5	14.6		14.3			14.	15.0	15.2		bright, broad, long
								4277.9		broad, bright, but uncertain as to character
								92.2		seems distinct
								97.3		very doubtful
	4304.1									faint
4313.7	14.5		13.5			13.9	13.4			faint
16.6							16.2			faint
								4335.3		very doubtful
								49.8		very doubtful
64.1	64.7		64.2			64.0	65.0	64.8		bright, rather long
71.0						70.6	71.9	71.8		not very bright
80.7	79.8					80.2	81.5	81.2		not very bright
							97.3	97.5		not very bright
								4402.3		bounded by absorption lines
						4411.7	12.4	12.6		not very bright
								19.2		bounded by absorption lines
								30.6		bounded by absorption lines
								52.1		bounded by absorption lines
								86.0		doubtful, diffused
								4537.4		sharp edge on side of smaller wave-length
								4588.9		faint, effect of contrast?
				71.1			4670.2	4670.8		bright
4677.8	78.2	77.7	77.6	78.0		78.0	76.3	77.1	77.6	very bright, well defined
84.4	83.5	84.4	84.4	84.9		84.7	84.0	84.1	84.5	very bright, well defined
96.7	96.7	96.3	96.6	97.1	98.3	97.6	96.9	96.7	97.2	very bright, well defined
							4705.0			faint
4714.7	15.1	14.7	14.4	14.6	16.0	15.3	14.4	14.5	14.2	very bright, well defined
							19.7			faint
							25.1			faint
36.2	36.5	36.2	36.3	36.2	37.8	4737.2	36.8	36.2	36.5	very bright, well defined
						5130				very faint
5165.7					65.4	5165.3	64.2	63.5		bright, well defined
					5539.5					very bright, well defined
					83.7					very bright, well defined
					5634.1					very bright, well defined

CYANOGEN SPECTRA

No. of Values and Mean Value	Kayser and Runge's Arc Spectra		Bright Lines Observed in Spectrum of Comet *b* 1893	In Comet *b* 1894	In Comet *a* 1899
1, 3855	3855.06	4th edge, 3d Cyanogen Band			
1, 3862	3861.86	3d edge, 3d Cyanogen Band			
2, 3867.6					
3, 3870.5	3871.54	2d edge, 3d Cyanogen Band	3870.0	3869.4	3870
3, 3874.1	3874.32	strong line, 3d Cyanogen Band			
2, 3877.1					
6, 3879.7	3879.62	mean of 6 strong lines, 3d CN Band			
10, 3882.3	3883.55	1st edge, 3d Cyanogen Band	3881.2	3879.5	3880
1, 3890					
2, 3990			3988	3987	3987
3, 4003					
1, 4014			4011	4015.7	4014
3, 4020.0			4019.2		4019
4, 4039.0				4040.9	
5, 4042.8			4043.1		4042
5, 4050.9	4051.00	strong line, 2d CN Band		4050.5	
3, 4052.9	4053.35	strong line, 2d CN Band	4052.3		4052
2, 4065.3					
3, 4068.8	4069.33	strong line, 2d CN Band	4069	4070.7	
4, 4074.0	4073.69	strong line, 2d CN Band	4075		4074
1, 4166	4165.34	mean of 2 strong lines, 2d CN Band			
1, 4169	4167.77	4th edge, 2d Cyanogen Band			
1, 4180	4180.98	3d edge, 2d Cyanogen Band			
1, 4185	4185.44	mean of 2 strong lines, 2d CN Band			
1, 4190	4189.63	mean of 3 strong lines, 2d CN Band			
2, 4193.2	4193.03	strong line, 2d Cyanogen Band	4196.2	4196.7	
6, 4214.6	4216.12	1st edge, 2d Cyanogen Band	4214.2	4213.8	421–
1, 4278					
1, 4292					
1, 4297			4298.4	4299.1	
1, 4304					
5, 4313.8			4313.0	4312.6	4313
2, 4316.4					
1, 4335			4334.8	4333.3	
1, 4350			4350	4350.6	
6, 4364.5	4365.01	3d edge, 5th Carbon Band	4366.2	4366.6	
4, 4371.3	4371.31	2d edge, 5th Carbon Band		broad band	4369
5, 4380.7	4381.93	1st edge, 5th Carbon Band		broad band	
2, 4397.5					
1, 4402					
3, 4412.2					
1, 4419					
1, 4440					
1, 4452					
1, 4486					
1, 4537					
1, 4589					
3, 4670.7					
9, 4677.6			4675.1		
9, 4684.3	4684.94	4th edge, 4th Carbon Band	4683.2	4681	
10, 4697.0	4697.57	3d edge, 4th Carbon Band	4697.0	4697.4	
1, 4705					
10, 4714.9	4715.31	2d edge, 4th Carbon Band	4716.0	4716.0	472–
1, 4720					
1, 4725					
10, 4736.6	4737.18	1st edge, 4th Carbon Band	4736.2	4735.8	
1, 5130	5129.36	2d edge, 3d Carbon Band	5126	5124	
5, 5164.8	5165.30	1st edge, 3d Carbon Band	5162.6	5163.5	
1, 5530.5	5540.86	3d edge, 2d Carbon Band			
1, 5583.7	5585.50	2d edge, 2d Carbon Band	5580		
1, 5634.1	5635.43	1st edge, 2d Carbon Band	5633	563–	

polarization effects in the parts of the tail near the head and in the coma, prior to August 14, but none after that date; and none were observed at any time in the light of the nucleus. The proportion of inherent to foreign light evidently increased so that by August 14 the comet's own radiations were strong enough to mask any polarization produced in diffused sunlight by the coma and tail. The particles composing the nucleus and that part of the tail yielding a continuous spectrum were evidently large in comparison with the wave-length of light. To move away from the sun in obedience to radiation-pressure, they would have to be smaller than certain limiting sizes. The angle at the comet between sun and earth was favorable for polarization effects, being about 80° for the August observations.

LICK OBSERVATORY

ON A NEW LAW OF SERIES SPECTRA

By W. RITZ

This communication is intended to show how we may derive from the known spectral series of an element, new series which represent accurately, without the inclusion of any new constant, nearly all of the series and lines recently discovered by Lenard, Konen, Hagenbach, Saunders, Moll, Ramage, and Bergmann. The new principle of combination also finds application to other spectra, particularly to helium and the earth alkalies. Closer relationships to the atomic weight than have been known hitherto are also furnished.

In its most convenient form, the series formula proposed by me reads:[1]

$$\nu = A - \frac{N}{[m+\alpha+\beta(A-\nu)]^2}, \tag{1}$$

where ν is the wave-number referred to a vacuum, N is a universal constant, m the numeral, and A the limit of the series; α and β are constants. For small values of m, ν may become negative, which must be taken into account in the term $\beta(A-\nu)$. The constants α and β characterize the course of the series: they are identical for two series with constant differences.

We also have approximately

$$\nu = A - \frac{N}{\left[m+\alpha+\frac{\beta N}{m^2}\right]^2}. \tag{2}$$

We use for abbreviation

$$m, \alpha, \beta = \frac{N}{[m+\alpha+\beta(A-\nu)]^2}. \tag{3}$$

Let the constants α and β have the values d and δ for the pairs of the first subordinate series for instance ("diffuse series" according to Rydberg's notation); for the second subordinate series (Rydberg's "sharp series") let the values be s, σ; for the principal series, let

[1] *Annalen der Physik*, **12**, 264, 1903. Inaugural Dissertation.

the values be p_1, π_1, and p_2, π_2; for the alkalies $\pi_1 = \pi_2$ very approximately.[1]

If the first subordinate series has a satellite, a different set of values, d', δ', applies to the second principal line, where again $\delta = \delta'$, approximately.

The statement of three series may now be written (*loc. cit.*, p. 291): Principal series:

$$\pm\nu = (1.5, s, \sigma) - (m, p_i, \pi_i), \qquad i = 1, 2;\ m = 2, 3, 4 \ldots, p_1 > p_2.$$

Second subordinate series:

$$\pm\nu = (2, p_i, \pi_i) - (m, s, \sigma), \qquad i = 1, 2;\ m = 1.5,\ 2.5,\ 3.5 \ldots\ldots$$

First subordinate series:

$$\pm\nu = \begin{cases} (2, p_i, \pi_i) - (m, d, \delta), & m = 3, 4, 5 \ldots \text{ (first principal line and satellite).} \\ (2, p_2, \pi_2) - (m, d', \delta'), & m = 3, 4, 5 \ldots \text{ (second principal line).} \end{cases}$$

The constant separation of the doublet is $\nu_1 = (2, p_1, \pi_1) - (2, p_2, \pi_2)$. On the basis of the observed data now available, the following points may be shown:[2]

1. In the equation of a principal series, if we replace 1.5 by the larger numbers 2.5, 3.5, ... ; and in the equation of a subordinate series, if we replace 2 by the larger numbers 3, 4, 5, ... , new lines will result which have been observed in many cases. This was already suspected by Rydberg.

2. For every symbol (m, α, β) there exists a minimum number m (namely, 3 for the first subordinate series; 1.5 for the second subordinate series, according to the notation of my dissertation already cited), which is a fundamental number; if we assign still smaller values to m, we should expect a stronger line, but in practice this line has not been observed. I have already shown[3] that in no spectrum does an actual line correspond to the numeral $n = 2$ of the first

[1] *Loc. cit.*, p. 291.

[2] The reader will find further particulars in a paper to appear presently in the *Physikalische Zeitschrift.*

[3] *Annalen der Physik*, **25**, 660, 1908; *Physikalische Zeitschrift*, **9**, 244, 1908.

subordinate series, in so far as the observations are adequate on this point.

3. If we form from these symbols the new combination $(1.5, s, \sigma)-(3, d, \delta)$ we obtain new lines which have been observed in case of *He*, *K*, *Rb*, but which are lacking in *Na* and *Li* thus far; for the earth alkalies they fall in the infra-red. The observed value for *He* is $\nu=26244.86$, the observed value is 26244.78.

4. In case of *Li* and *Na*, there has been also observed the combination $(2, p_1\pi_1)-(m, p_1\pi_1)$; $(2, p_2\pi_2)-(m, p_2\pi_2)$; $m=3, 4 \ldots$.

5. If we form $(3, d, \delta)-(m, p_1-p_2, \pi_1-\pi_2)$; $m=4, 5, \ldots$. then we get the infra-red series found by Bergmann[1] for *K*, *Rb*, *Cs*, and the corresponding lines for *Li*, *Na*, *He*, which were unknown.

For *Rb* and *Cs*, the first subordinate series has a satellite, so that we have further a second series $(3, d', \delta')-(m, p_1-p_2, \pi_1-\pi_2)$, which runs along in the neighborhood of the first, and has a constant difference with respect to this, as was observed by Bergmann, and even earlier by Saunders in case of *Cs*, while for *Rb* the lines could not be separated. The fundamental number for this newly formed symbol $(m, p_1-p_2, \pi_1-\pi_2)$ is $m=4$.

6. The following combinations also exist:

$$(2, p_i, \pi_i)-(m, p_1-p_2, \pi_1-\pi_2), \quad i=1, 2; \; m=4, 5, \ldots.$$

The pair shading toward the violet of *Na*, the corresponding pair of *Cu*, and the series of lines which have been found in *Li* and *Na*, by Lenard, Konen, and Hagenbach, belong here.

$$(4, p_1-p_2, \pi_1-\pi_2)-(m, p_1-p_2, \pi_1-\pi_2).$$

7. Extended investigations, such as exist for the alkalies, are lacking for the earth-alkalies in the infra-red region; and the principal series have not been observed, showing that the test of the principle of combination, in the sense hitherto used, cannot be made at present. The following circumstances, however, indicate that it is also valid here: in the first subordinate series of the series of triplets, with two satellites, we have to introduce the new symbols (m, p_3, π_3), (m, d'', δ'') with $p_1>p_2>p_3$. In the first approximation we may neglect

[1] Inaugural Dissertation, Jena, 1907; also C. Runge, *Physikalische Zeitschrift*, 9, 1, 1908.

$\pi_1-\pi_2$, $\pi_2-\pi_3$, and compute p_1-p_2, p_2-p_3, from the limits of the subordinate series, in spite of the fact that the principal series have not been observed; p_1-p_2 will be about twice as large as p_2-p_3, and the two differences of the vibration number (designated by Rydberg as ν_1 and ν_2) are

$$\nu_1=(2, p_2, \pi_2)-(2, p_1, \pi_1)\ ; \quad \nu_2=(2, p_3, \pi_3)-(2, p_2, \pi_2)\ .$$

There frequently occur double lines, with the distance ν_1, and triplets with the distance ν_1, ν_2, outside of the series. This is to be expected on the principle of combination only, and only when $(2, p_i, \pi_i)$ are associated with any quantity (m, α, β). These doublets and triplets must therefore belong to series which end at the same point $(2, p_i, \pi_i)$ as the subordinate series already known, but of which only a few terms of sufficient intensity are to be perceived.

It was further possible to find the equation of the second subordinate series for the strong ultra-violet pairs for *Ca*, *Sr*, *Ba*, for which the difference of the vibration-numbers ν' is to be twice as large as ν_1, and from the elements of these double lines to form the corresponding differences of the constants p, on the assumption that $\pi_1-\pi_2=0$. The distance differs from that of the series of triplets by only about 15 per cent.; in the same exact computation, the two quantities would probably become equal to each other, which would correspond to the principle of combination.

Finally, subordinate series of close triplets have recently been found for *Ca* and *Sr*, by Fowler and by Saunders, from the limits of which again the differences $p'_1-p'_2$, $p'_2-p'_3$, can be formed. Within the limits of accuracy of these, it appears that they are the same as those computed from the satellites and principal lines of the first subordinate series $d-d'$, $d'-d''$, so that a new combination exists here.

It further appears of advantage, in respect to the relations with the atomic weight, to introduce the constants of the formula in place of the wave-numbers, in spite of the fact that this can only be regarded as an approximation. It is well known, for instance, that the quotient of ν_1 and the square of the atomic weight μ does not vary much within a group of chemically related elements; but on the contrary it varies greatly from group to group, from 31.6 for *Cs*,

to 187.0 for *Tl*. We introduce $\frac{p_2-p_1}{\mu^2}$ in place of $\frac{\nu_1}{\mu^2}$ and the variation becomes much less.

The discrepancies which still remain are probably due in part to the incompleteness of the series formulae, but particularly to the neglect of $\pi_1-\pi_2$, which certainly is not admissible in the case of *Al*, *In*, and *Tl*. A knowledge of the principal series of these elements would enable us to decide about this.

Element*	*Na*	*K*	*Rb*	*Cs*	*Cu*	*Ag*	*Mg*	*Ca*
$\frac{\nu_1 10^3}{\mu^2}$	32.3	37.8	32.3	31.6	61.8	79.0	68.8	66.1
$\frac{p_2-p_1}{\mu^2}10$....	14.2	18.9	18.0	18.6	18.6	24.2	14.6	17.7

Element*	*Sr*	*Zn*	*Cd*	*Hg*	*Al*	*In*	*Tl*	*He*
$\frac{\nu_1 10^3}{\mu^2}$	51.5	91.0	93.2	115.4	152.8	172.1	187.0	63.8
$\frac{p_2-p_1}{\mu^2}10$....	15.7	17.2	18.6	22.3	24.8	29.2	32.7	20.4

* According to the summary by Rydberg, *Rapports du Congrès de Physique*, Paris, 1900. Tome 2.

It is known that with increasing atomic weight the series fall off with increasing rapidity, so that only those lines are observed which correspond to the lowest numbers of the order. At the same time the number of the different series increases (even *Mg* shows series of double lines and of simple lines in addition to the series of triplets) and the number of combinations increases. Thus we finally reach a spectrum in which we may perceive a large number of characteristic constant differences, but no series. It seems to me very probable that the so-called spectra of the second class which exhibit this behavior are spectra with very many combinations and with very slightly developed series.

The computation of a quantity (m, α, β) assumes a knowledge of the elements of a series in the formula of which it enters. This element may be determined with great accuracy, except when only a few diffuse lines are observed; it is also somewhat independent of the outstanding uncertainty as to the exact form of the equation

of the series. In the cases cited under (1), (3), (4), we have indeed to compute exclusively differences and sums of observed wave-numbers; the accuracy is naturally still greater here. But on the other hand the constants entering into the symbol $(m, p_1-p_2, \pi_1-\pi_2)$ may be varied by 20 per cent. without making the difference between computation and observation much worse. On account of their smallness the influence of the constants p_1-p_2, $\pi_1-\pi_2$, is relatively slight compared to that of m, and the series run along nearly parallel with Balmer's series, particularly for *Li*, *Na*, and *He;* for greater atomic weight the observations are hitherto too inaccurate to give us any certainty on this point.

In a paper which recently appeared[1] I have shown that we may cite systems of the simplest sort of which the energy is purely electromagnetic and which observe Balmer's formula, the laws of series and the analogous Zeeman effect, etc. The vibration

$$\nu = N\left[\frac{1}{4} - \frac{1}{m^2}\right]$$

is produced by the magnetic field of $m-2$ elementary magnets turned toward each other, which are identical among themselves: in addition to this magnetic series the electron is subject to only rigid combinations. Similar facts hold good for other spectra. Higher numbers of the order thus correspond in a certain degree with higher magnetic polymerizations, which constantly become less stable, so that the lines become broader with an increasing number of the order, and also become constantly weaker. Violent motions, such as occur in an electric spark, are also unfavorable to stability, whence the series in the spark cease at lower numbers of the order than in the arc.

The magnetic field in an atom may be regarded in all spectra as produced by two poles of opposite sign, which separately may occupy different positions in the atom. In case of hydrogen, these points lie at equal distances on a straight line. It would appear that we may more generally state the principle that the simple lines refer to or depend upon the positions of these poles in the atom. In $(m, \alpha, \beta) - (n, \alpha', \beta')$ each of the terms represents the influence of one pole;

[1] "Magnetische Atomfelder und Serienspektren," *Annalen der Physik*, 25, 660, 1908.

and, as we have shown before, the possible positions of the separate poles permit the most varied combinations of the poles in pairs.

As to a certain minimum prescribed in the number of the order m—which occurs in the case of no known processes of vibration—this mode of representation only affirms that the magnet poles are held by the structure of the atom at a certain minimum distance from the electron vibrating within an atom, an idea which is quite plausible.

GÖTTINGEN
June 1908

THE PASADENA LABORATORY OF THE MOUNT WILSON SOLAR OBSERVATORY[1]

By GEORGE E. HALE

The spectroscopic laboratory erected in 1905 on Mount Wilson was described in *Contributions from the Solar Observatory*, No. 10.[2] As stated in that paper, our investigations of sun-spot spectra made it necessary to supplement the equipment provided on Mount Wilson with a large electric furnace, which was installed in the Pasadena instrument shop. As the further development of our sun-spot work demanded the use of a more perfect electric furnace and as our apparatus required more current than could be economically generated on Mount Wilson, it seemed advisable to take advantage of the opportunity afforded in Pasadena to obtain electrical energy, at moderate cost, from the Edison Company. Accordingly a small laboratory, adjoining our instrument shop and standing immediately in front of the Hooker Building, was erected during the winter of 1908 (Plate XVI).

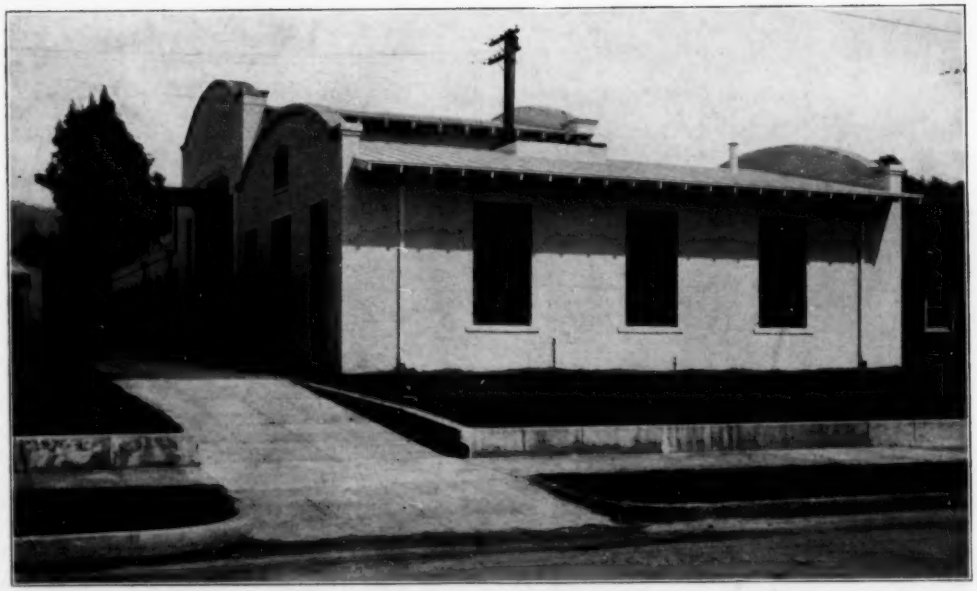
In the Mount Wilson laboratory the various light-sources are arranged on the circumference of an annular pier. A plane mirror at the center, which can be rotated about a vertical axis, reflects the rays from the light-source under examination to a concave mirror, which forms an image on the slit of a horizontal Littrow spectrograph of 18 feet (5.5 m) focal length. In the Pasadena laboratory, profiting by experience with the tower telescope,[3] a vertical Littrow spectrograph, of 30 feet (9.1 m) focal length, is mounted in a well, with waterproof brick walls, extending 30 feet below the surface of the ground. The electric furnace and other light-sources stand on separate piers, arranged in a circle about the center of the spectrograph slit. The spectrograph can be rotated about the axis of the

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 27.

[2] *Astrophysical Journal*, **24**, 61, 1906.

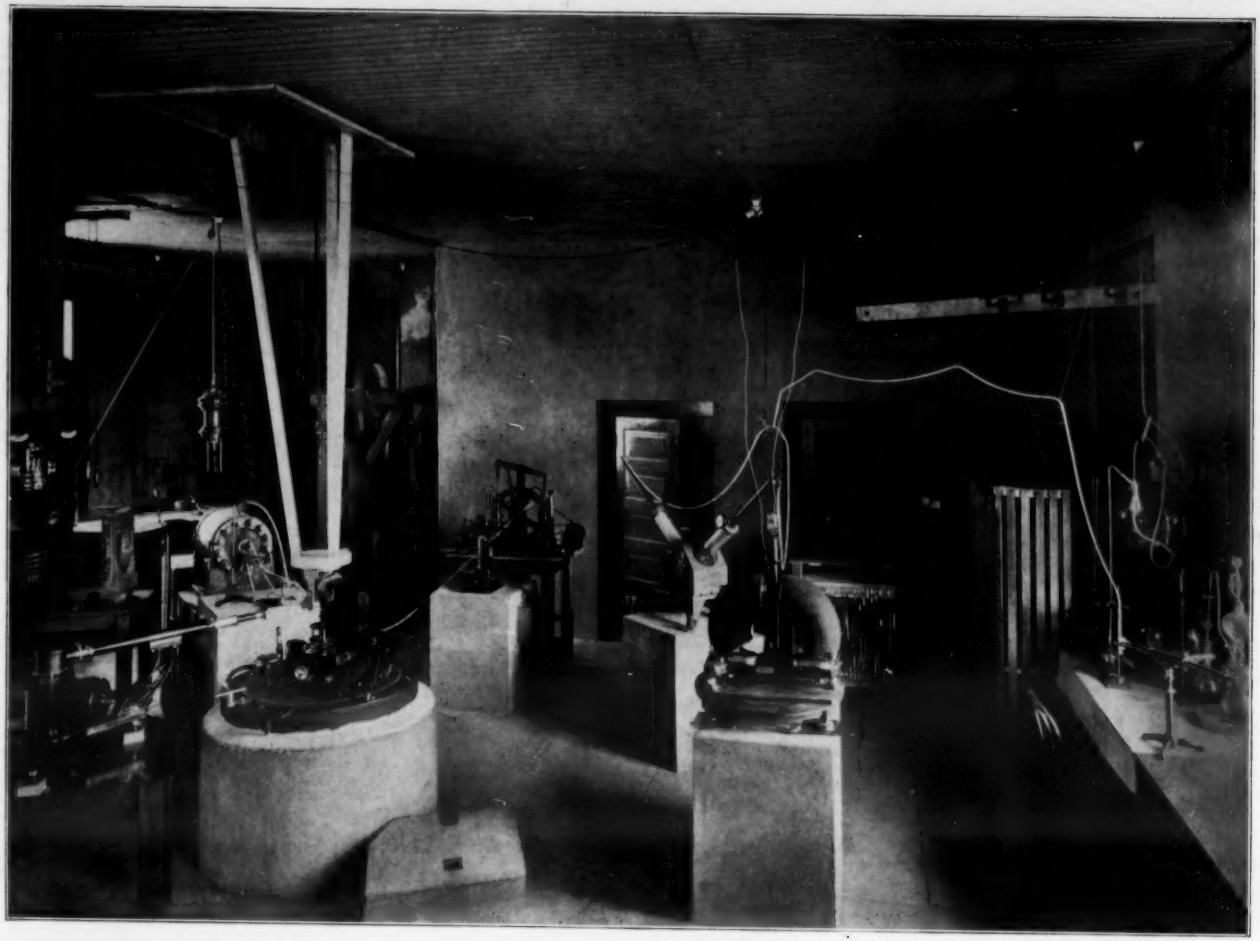
[3] *Contributions from the Mount Wilson Solar Observatory*, No. 23; *Astrophysical Journal*, **27**, 204, 1908.

PLATE XVI



THE PASADENA LABORATORY

PLATE XVII



INTERIOR OF THE PASADENA LABORATORY

collimator and light from any source is reflected into the slit by means of a plane mirror standing (at 45°) above it (Plate XVII).[1]

This arrangement determines the general plan of the laboratory (Fig. 1). The outside dimensions of the building are 32×44 feet. The walls are of brick, the floor of cement, and the ceiling of corrugated iron. The well which contains the spectrograph, 8½ feet inside diameter, is near the middle of the principal room. As the spectrograph stands eccentrically, near one side of the well, considerable space is left in the well for other instruments requiring constant temperature conditions.

Except in one particular, the 30-foot spectrograph is precisely similar to the one used with the tower telescope.[2] In addition to an 8-inch (20.3 cm) objective of 30 feet focal length, it is supplied with a 5-inch (12.7 cm) objective of 13 feet (4 m) focal length.[3] This objective, together with an adjustable grating-holder mounted in conjunction with it, can be swung out of the axis of the spectrograph when the objective of 30 feet focal length is to be employed. Thus a considerable range of dispersion, from the first-order spectrum with the 13-foot objective to the fourth-order spectrum with the 30-foot objective, is available. Both objectives can be focused from the eye-end of the instrument and the grating can be rotated from the same point. The only gratings at present available are a 5-inch Rowland plane, having 14,438 lines to the inch, kindly loaned to us by the Johns Hopkins University, and a 4-inch Michelson plane, having 500 lines to the millimeter.

The concrete floor is continued over the well, the spectrograph ring being supported on a cylinder of concrete rising from it. The temperature at the bottom of the well is so constant that exposures of any desired length can be given, without fear of displacement of the lines arising from changes in the temperature of the grating.

A small fireproof room in the laboratory contains five transformers,

[1] The mirror support shown is a temporary one, and will be replaced later by a different apparatus, carrying also a lens, on a radial arm, to form an image of any source on the slit.

[2] *Loc. cit.*

[3] Both visual and photographic objectives of this size, formerly employed in photographing spectra with the Snow telescope, are available for use with this spectrograph.

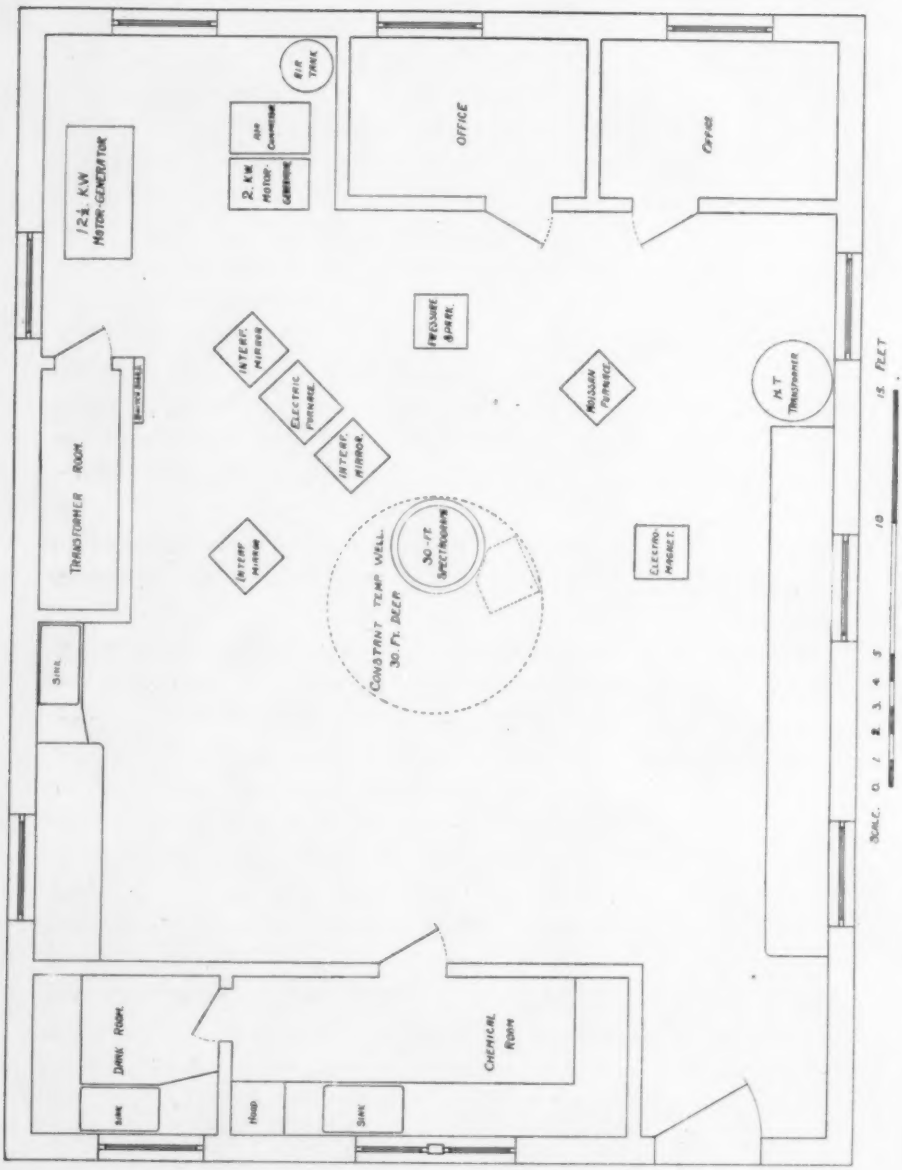


FIG. 1.—Plan of Laboratory.

connected with the 2000-volt alternating current circuit of the Edison Electric Company, as follows:

a) One low-voltage transformer, formerly used for our experiments on fused quartz, having a capacity of 50 K. W., with connections for 5, 10, 20, or 30 volts. By means of very heavy copper cables, passing through a conduit beneath the floor, this transformer supplies the resistance-tube electric furnace with current.

b) Two 30 K. W. transformers, primarily for heavy-current arc work. These may be connected either in series or parallel, giving 52 or 104 volts, with a capacity of 60 K. W., or 208 volts, with a capacity of 30 K. W.

The secondary terminals of transformers *a*) and *b*) are mounted on a slate pier in the transformer room, where they may be joined by heavy copper lugs to cables passing through conduits to the three piers designed for furnace and arc work. Thus the voltages above mentioned, ranging from 5 to 208, are available as desired at any one of these piers. The three transformers are controlled by primary oil switches, operated by cords from without the transformer room.

c) Two 15 K. W. transformers, which supply power to the machinery of the instrument and optical shops, to the motors that drive the direct-current generators in the laboratory, to the high-voltage transformer, etc.

The 5 K. W. high-voltage transformer, which stands on the opposite side of the room, is connected with highly insulated overhead wires passing across the laboratory, from which leads may be dropped to any of the piers where a spark is to be used. This transformer contains a series of step-up connections, giving 1000, 2000, 4000, 8000, 16,000, 32,000, or 64,000 volts at the secondary terminals. Within the inclosure which surrounds the transformer there are a series of self-induction coils and a large condenser, consisting of alternate plates of sheet metal and plate glass immersed in oil.

Direct current is supplied from two sources:

a) A 12½ K. W. dynamo, direct connected with a three-phase motor. Both of these machines stand on a heavy concrete pier, separated from the floor and resting on a bed of sawdust. In this way the vibration is so greatly reduced that it is not perceptible in

the 30-foot spectrograph. By regulating the field of the motor the dynamo gives voltages varying from 30 to about 120. Thus the Dubois electro-magnet, which is intended for use at 64 volts, can be excited without a series rheostat. The high voltage is mainly employed for powerful electric current arcs and other similar purposes.

b) A 2 K. W. generator, direct connected with a three-phase motor, both standing on a pier separated from the floor. This gives direct-current voltages ranging from 90 to 120, and serves well for small arcs and other apparatus requiring moderate currents. The motor is also used to drive an air-compressor built by Cook of Manchester, after a design kindly prepared for us by Mr. Petavel.

Both dynamos are joined to the switchboard, where they may be connected to wires passing through conduits to two of the piers.

The principal light-sources and auxiliary instruments now employed in the laboratory are as follows:

a) A carbon or graphite tube resistance furnace (on the left of Plate XVII), inclosed in a steel cylinder capable of withstanding pressures up to 200 atmospheres. This furnace, which was designed by Dr. King, is described by him in another article.[1] The highest temperature hitherto attained in it, as measured with a Wanner pyrometer, is 3015° C. It has thus served admirably for the study of the spectra of such refractory metals as titanium and vanadium, permitting the relative intensities of their lines to be recorded at widely different temperatures. This furnace is also intended for investigations of anomalous dispersion, in conjunction with a Michelson interferometer and the 30-foot spectrograph.

b) A rotating arc in a pressure chamber, formerly used in the Mount Wilson laboratory.

c) An inclined arc electric furnace (near the middle of Plate XVII), similar in type to one used by Moissan, but modified according to designs by Dr. Olmsted so as to permit the arc to be observed in an atmosphere of hydrogen or other gas. For regulating the current a large rheostat is provided. This furnace is now used by Dr. Olmsted in his work on the fluted spectra of calcium hydride and other compounds found in the spectra of sun-spots and red stars. A Geryk

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 28.

duplex vacuum pump, driven by a small electric motor, is used with the two furnaces when low pressures are required.

d) Spark terminals, mounted between the poles of a large DuBois electro-magnet formerly used in the Mount Wilson Laboratory. This apparatus, which is shown on the right of Plate XVII, is being used for the study of the Zeeman effect in the spectra of iron, titanium, and other elements that occur in the spectra of sun-spots.

e) An ordinary electric arc, used for comparison spectra, etc.

A one-prism quartz spectrograph and a direct vision spectroscope are employed for the preliminary examination of spectra. Other apparatus used in the Mount Wilson Laboratory and described in *Contribution* No. 10 is also available. A two-mirror heliostat, mounted on the roof immediately above the 30-foot spectrograph, supplies sunlight for comparison spectra. Piers for vacuum tube apparatus and other light-sources will be erected as occasion demands.

At the west end of the building there is a small chemical laboratory and a photographic dark-room. At the east end are the offices of Dr. King, superintendent of the Physical Laboratory, and Dr. Olmsted.

A more detailed account of the instruments used in the laboratory will appear in subsequent papers.

MOUNT WILSON SOLAR OBSERVATORY
August 1908

ASTRONOMICAL AND ASTROPHYSICAL SOCIETY

The Astronomical and Astrophysical Society of America held its meeting at Hotel Victory, Put-in-Bay Island, Lake Erie, on August 25, 26, and 27, with Professor E. C. Pickering, president, in the chair.

The following papers were presented:

ASAPH HALL: "Formulas Used for the Reduction of Satellite Observations."

G. W. HOUGH: "Doolittle's Measures of the Hough Double Stars."

W. S. EICHELBERGER: "The Standard Clock of the U. S. Naval Observatory."

C. L. DOOLITTLE: "Examination of the Reflex Zenith Tube."

E. E. BARNARD: "On the Period of the Variable Star No. 33 in Messier 5, and on the Constancy of the Period."

E. D. ROE, JR.: "Achromatic and Apochromatic. Comparative Tests. Preliminary Communication."

E. E. BARNARD: "On a Quick Visual Method of Correcting for the Changes of Focus in the Large Visual Telescope when Used for Photography with a Color-Filter."

E. E. BARNARD: "On the Focal Changes in the Image of *Nova Persei* and on the Focus for Some of the Wolf-Rayet Stars."

G. C. COMSTOCK: "Approximate Ephemerides of the Fixed Stars."

E. C. PICKERING: "A New Form of Stellar Photometer."

E. C. PICKERING: "Standard Photographic Magnitudes"

FRANK SCHLESINGER: "On the Character of the Light-Variations of *u 68 Herculis*."

J. A. PARKHURST: "Light-Curve of the Variable Star *SU Cassiopeiae* from Extra-Focal Photographs." (Lantern.)

E. E. BARNARD: "On the Inequalities in the Proper Motion of the Star *Krueger 60*, Due to Orbital Motion." (Lantern.)

J. S. PLASKETT: "The Coelostat Telescope of the Dominion Observatory." (Lantern.)

J. S. PLASKETT: "Camera Objectives for Spectrographs." (Lantern.)

MRS. M. FLEMING: "A Proposed Sixth Type of Stellar Spectra."

C. D. PERRINE: "A Determination of the Solar Parallax from the Crossley Photographs of *Eros*."

W. W. CAMPBELL: "The Lick Observatory Eclipse Expedition to Flint Island."

PHILIP FOX: "Prominences on the Solar Disk." (Lantern.)

MILTON UPDEGRAFF: "The Work of the Nautical Almanac Office."

G. W. HOUGH: "On an Infinite Universe."

G. C. COMSTOCK: "The Luminosity of the Brighter Lucid Stars."

FRANK SCHLESINGER: "Photographic Determinations of Stellar Parallax with the Yerkes Refractor."

JOEL STEBBINS: "The Measurement of Starlight with a Selenium Photometer."

EDWIN B. FROST: "Spectrographic Observations."

F. H. SEARES: "Results of Photometric Investigations."

W. T. CARRIGAN: "An Investigation of Terms in the Mean Longitudes of *Mars* and the Earth, That Have the Argument, 3 J−8 M+4 E″."

W. J. HUMPHREYS: "The Temperature Gradient of the Atmosphere and an Attempt to Account for the Upper Inversion."

J. S. PLASKETT: "Effect of Increasing the Slit-width upon the Accuracy of Radial Velocity Determinations."

RAYMOND S. DUGAN: "The *Algol* System *RT Persei*."

The time and place of the next meeting will be decided later by the Council.

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

Accuracy in the proof is gained by having manuscripts type-written, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

All drawings should be carefully made with India ink on stiff paper, usually each on a separate sheet, on about double the scale of the engraving desired. Lettering of diagrams will be done in type around the margins of the cut where feasible. Otherwise printed letters should be put in lightly with pencil, to be later impressed with type at the editorial office, or should be pasted on the drawing where required.

Where an unusual number of illustrations may be required for an article, special arrangements are made whereby the expense is shared by the author or by the institution he represents.

Authors will please carefully follow the style of this *Journal* in regard to footnotes and references to journals and society publications.

Authors are particularly requested to employ uniformly the metric units of length and mass; the English equivalents may be added if desired.

If a request is sent *with the manuscript*, one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

The editors do not hold themselves responsible for opinions expressed by contributors.

The ASTROPHYSICAL JOURNAL is published monthly except in February and August. The annual subscription price is $4.00; postage on foreign subscriptions 62 cents additional. Business communications should be addressed to *The University of Chicago Press, Chicago, Ill.*

All papers for publication and correspondence relating to contributions should be addressed to *Editors of the* ASTROPHYSICAL JOURNAL, *Yerkes Observatory, Williams Bay, Wisconsin, U. S. A.*